# Algebraic Local Cohomology Classes Associated with Semi-quasihomogeneous Singularities

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Session: Parametric polynomial computations

・We would like to make a software (or package) for studying and analyzing singularities.

・We would like to study singularity theory **by computers.**

# Today's talk

A new algorithm for computing bases of algebraic local cohomology classes associated with semi-quasihomogeneous singularities is introduced.
We generalize this algorithm to parametric cases.

In our previous works, we know the following;

- Algebraic local cohomology classes have a lot of properties for analyzing singularities.
- Standard bases are easily obtained from algebraic local cohomology classes.

# Contents

1. Algebraic local cohomology (review)
2. Semi-quasihomogeneous and Poincaré polynomials
3. An algorithm for computing algebraic local cohomology
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# Algebraic Local cohomology (review)

$K$ : $\mathbb{Q}$ or $\mathbb{C}$, $\quad x = x_1, \ldots, x_n$ : $n$ variables,

Let $H^n_{[O]}(K[x])$ denote the set of algebraic cohomology classes with coefficients in $K$, defined by

$$H^n_{[O]}(H[x]) := \lim_{k\to\infty} Ext^n_{K[x]}(K[x]/\langle x_1, x_2, \ldots, x_n\rangle^k, K[x])$$

where $\langle x_1, \ldots, x_n\rangle$ is an ideal generated by $x_1, \ldots, x_n$.

**Fact**

Let $X$ be a neighbourhood of the origin $O$ of $\mathbb{C}^n$. Consider the pair $(X, X - O)$ and its relative Čech covering. Then, any section of $H^n_{[O]}(K[x])$ can be represented as an element of relative Čech cohomology.

We use the notation $\sum c_\lambda \left[\frac{1}{x^{\lambda+1}}\right]$

for representing algebraic local cohomology classes in $H^n_{[O]}(K[x])$ where $c_\lambda \in K$, $x^{\lambda+1} = x_1^{\lambda_1+1} x_2^{\lambda_2+1} \ldots x_n^{\lambda_n+1}$ with $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n) \in \mathbb{N}^n$.

The multiplication is defined as

$$x^\alpha \left[\frac{1}{x^{\lambda+1}}\right] = \begin{cases} \left[\frac{1}{x^{\lambda+1-\alpha}}\right] & \lambda_i \geq \alpha_i, i = 1, \ldots, n, \\ 0 & \text{otherwise,} \end{cases}$$

where $\alpha = (\alpha_1, . , \alpha_n) \in \mathbb{N}^n$ and $\lambda+1-\alpha = (\lambda_1+1-\alpha_1, . , \lambda_n+1-\alpha_n)$.

**One can regard**

$2\left[\frac{1}{xy^2}\right] - \frac{4}{3}\left[\frac{1}{x^2y^3}\right]$ **as** $2x^{-1}y^{-2} - \frac{4}{3}x^{-2}y^{-3}$

$\xi = \xi_1, \ldots, \xi_n : n$ variables,

**To manipulate algebraic local cohomology classes efficiently on computer.**

| Čech representation | | polynomial representation |
|---|---|---|
| $\sum c_\lambda \left[ \dfrac{1}{x_1^{\lambda_1+1} x_2^{\lambda_2+1} \cdots x_n^{\lambda_n+1}} \right]$ | $\longleftrightarrow$ | $\sum c_\lambda \xi_1^{\lambda_1} \xi_2^{\lambda_2} \cdots \xi_n^{\lambda_n}$ |

where $c_\lambda \in K$, $\lambda = (\lambda_1, \ldots, \lambda_n) \in \mathbb{N}^n$.

**Example**

$$2\left[\frac{1}{x_1^3 x_2^4}\right] - \frac{4}{3}\left[\frac{1}{x_1^2 x_2^3}\right] \longleftrightarrow 2\xi_1^2\xi_2^3 - \frac{4}{3}\xi_1\xi_2^2$$

The multiplication for polynomial representation is defined as follows:

$$x^\alpha * \xi^\lambda = \begin{cases} \xi^{\lambda-\alpha} & \lambda_i \geq \alpha_i, i = 1, \ldots, n, \\ 0 & \text{otherwise,} \end{cases}$$

where $\alpha = (\alpha_1, \ldots, \alpha_n) \in \mathbb{N}^n, \lambda = (\lambda_1, \ldots, \lambda_n) \in \mathbb{N}^n$, and $\lambda - \alpha = (\lambda_1 - \alpha_1, \ldots, \lambda_n - \alpha_n)$. (We use " $*$ " for polynomial representation.)

**After here, we adapt polynomial representation to represent an algebraic local cohomology class.**

# Semi-qusihomogeneous

$x^\alpha := x_1^{a_1} x_2^{a_2} \cdots x_n^{a_n}$, $\mathbf{w} = (w_1, \ldots, w_n) \in \mathbb{N}^n$ : a weight vector for $x = (x_1, \ldots, x_n)$

**Definition(quasihomogeneous)**

- We define a **weighted degree** of $x^\alpha$, w.r.t. $\mathbf{w}$ by
$$|x^\alpha|_{\mathbf{w}} := \sum_{i=1}^n w_i \alpha_i.$$
- A nonzero polynomial $f \in K[x]$ is **quasihomogeneous of type** $(d; \mathbf{w})$ if all terms of $f$ have the same weighted degree $d$ w.r.t. $\mathbf{w}$, i.e.,
$$f = \sum_{|x^\alpha|_{\mathbf{w}} = d} c_\alpha x^\alpha \quad \text{where } c_\alpha \in K.$$
- We define a weighted degree of $f$ by
$$\deg_{\mathbf{w}}(f) := \max\{|x^\alpha|_{\mathbf{w}} : x^\alpha \text{ is a term of } f\}.$$

**Example** $\mathbf{w} = (7, 3)$ (for $(x, y)$)

$$f = \underbrace{x^3}_{21} + \underbrace{y^7}_{21},$$

quasihomogeneous of type $(21; (7, 3))$

$$g = \underbrace{x^3}_{21} + \underbrace{y^7}_{21} + \underbrace{2xy^5}_{22}$$

Not quasihomo.

## Definition(Semi-quasihomogeneous)

Let $f \in K[x]$ be a polynomial. We define $\mathrm{ord}_{\mathbf{w}}(f) = \min\{|x^\alpha|_{\mathbf{w}} : x^\alpha$ a term of $f\}$. The polynomial $f$ is called **semi-quasihomogeneous of type** $(d;\mathbf{w})$ if $f$ is of the form $f = f_0 + g$ where $f_0$ is a quasihomogeneous of type $(d;\mathbf{w})$ with an isolated singularity at the origin, $f = f_0$ or $\mathrm{ord}_{\mathbf{w}}(f - f_0) > d$.

**Example** $f = \underbrace{x^3}_{21} + \underbrace{y^7}_{21} + \underbrace{2xy^5}_{22}, \quad \mathbf{w} = (7,3) \ (\text{for } (x,y))$

semi-quasihomogeneous of type $(21;(7,3))$

## Definition(weighted term orders)

For two multi-indices $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)$ and $\lambda' = (\lambda'_1, \lambda'_2, \ldots, \lambda'_n)$ in $\mathbb{N}^n$, we denote

$$\xi^{\lambda'} \prec \xi^{\lambda} \text{ or } \lambda' \prec \lambda$$

if $|\xi^{\lambda'}|_{\mathbf{w}} < |\xi^{\lambda}|_{\mathbf{w}}$, or if $|\xi^{\lambda'}|_{\mathbf{w}} = |\xi^{\lambda}|_{\mathbf{w}}$ and there exists $j \in \mathbb{N}$ so that $\lambda'_i = \lambda_i$ for $i < j$ and $\lambda'_j < \lambda_j$.

In this talk, we fix $\prec$ as a weighted term order.

$h$: an algebraic local cohomology class (in polynomial rep.)

$$h = c_\lambda \xi^\lambda + \sum_{\lambda' \prec \lambda} c_{\lambda'} \xi^{\lambda'},\ c_\lambda \neq 0$$

$\xi^\lambda$: the **head term**(written $\mathrm{ht}(h)$),

$\xi^{\lambda'}$: the **lower terms**.

$$h = \underbrace{\sum_{|\xi^\lambda|_{\mathbf{w}}=a} c_\lambda \xi^\lambda}_{\text{the head part}} + \underbrace{\sum_{|\xi^{\lambda'}|_{\mathbf{w}}<a} c_{\lambda'} \xi^{\lambda'}}_{\text{the lower part}}$$

# Poincaré polynomial (key)

**Definition** $\mathbf{w} = (w_1, \ldots, w_n) \in \mathbb{N}^n$ : a weight vector

The Poincaé polynomial of type $(d;\mathbf{w})$ is defined by

$$P_{(d;\mathbf{w})}(t) = \frac{t^{d-w_1}-1}{t^{w_1}-1} \cdot \frac{t^{d-w_2}-1}{t^{w_2}-1} \cdots \frac{t^{d-w_n}-1}{t^{w_n}-1}$$

**Example**

A polynomial $f = x^3y + xy^4$ is quasihomogeneous of type $(11;(3,2))$.

$$P_{(11;(3,2))}(t) = \frac{t^{11-3}-1}{t^3-1} \cdot \frac{t^{11-2}-1}{t^2-1} = 1+t^2+t^3+t^4+t^5+2t^6+t^7+t^8+t^9+t^{10}+t^{12}.$$

# The purpose of this talk

$f = f_0 + g$ : a semi-quasihom. poly. of type $(d;\mathbf{w})$

$f_0$ : a quasihomo. poly. of type $(d;\mathbf{w})$ and defines an isolate singularity at the origin

We define a vector space $H_f$ to be the set of algebraic local cohomology classes, in polynomial representation, in $K[\xi]$ that are annihilated by Jacobi ideal $\langle \frac{\partial f}{\partial x_1}(x), \ldots, \frac{\partial f}{\partial x_n}(x) \rangle$,

$$\boxed{H_f := \left\{ h \in K[\xi] \,\middle|\, \frac{\partial f}{\partial x_1}(x) * h = \frac{\partial f}{\partial x_2}(x) * h = \cdots = \frac{\partial f}{\partial x_n}(x) * h = 0 \right\}.}$$

**In this talk, an algorithm for computing a basis of $H_f$ , is introduced.**

$(\dim(H_f) < \infty)$

# The first result

$$P_{(d;\mathbf{w})}(t) = \sum_{i=1}^{\ell} m_i t^{d_i} \text{ : the Poincaré polynomial of type } (d;\mathbf{w}) \text{ where } m_i \in \mathbb{N},$$

$$D_P := \bigcup_{i=1}^{\ell} \{\underbrace{d_i, d_i, \ldots, d_i}_{m_i \text{ elements}}\}$$

**Example** $P_{(11;(3,2))}(t) = 1 + t^2 + t^3 + t^4 + t^5 + 2t^6 + t^7 + t^8 + t^9 + t^{10} + t^{12}.$

$$D_P = \{0, 2, 3, 4, 5, 6, 6, 7, 8, 9, 10, 12\}$$

$Q$ : a set in $K[\xi]$, $\quad D_Q := \{\deg_{\mathbf{w}}(q) \in \mathbb{N} \mid q \in Q\}$

**Theorem(The 1st result)** $f_0$ : a quasihomo. poly. of type $(d;\mathbf{w})$

There exists a basis $Q$ of $H_{f_0}$ which satisfies the following conditions

(1) $Q$ consists of quasihomogeneous polynomials.

(2) $\boxed{D_Q = D_P.}$

**Theorem(The 1st result)** $f_0$ : a quasihomo. poly. of type $(d;\mathbf{w})$

There exists a basis $Q$ of $H_{f_0}$ which satisfies the following conditions

(1) $Q$ consists of quasihomogeneous polynomials.

(2) $\boxed{D_Q = D_P.}$

**Find a quasihomo. poly. $h$ whose weighted-degree belongs to $D_Q$. Check $\partial f_0/\partial x_i * h = 0$.**

**A basis of $H_{f_0}$ is easily computed by the theorem.**

**Outline**

**The algorithm consists of the following two part.**

**(1) Compute a basis *Q* of $H_{f_0}$ by using a Poincaré polynomial.**

**(2) Compute a basis of $H_f$ by using the result *Q*.**

Example $f = x^3 + xy^5 + 2y^8, \quad \mathbf{w} = (5,2)$

$f_0 = x^3 + xy^5, \quad g = 2y^8, \quad d = \deg_{\mathbf{w}}(f_0) = 15$

We compute a basis $Q$ of $H_{f_0} = \left\{ h \in K[\xi,\eta] \,\middle|\, \frac{\partial f_0}{\partial x} * h = \frac{\partial f_0}{\partial y} * h = 0 \right\}.$

$$P_{(15;\mathbf{w})}(t) = \frac{t^{15-5}-1}{t^5-1} \cdot \frac{t^{15-2}-1}{t^2-1} = 1 + t^2 + t^4 + t^5 + t^6 + t^7 + t^8 + t^9 + t^{10} + t^{11} + t^{12} + t^{14} + t^{16}$$

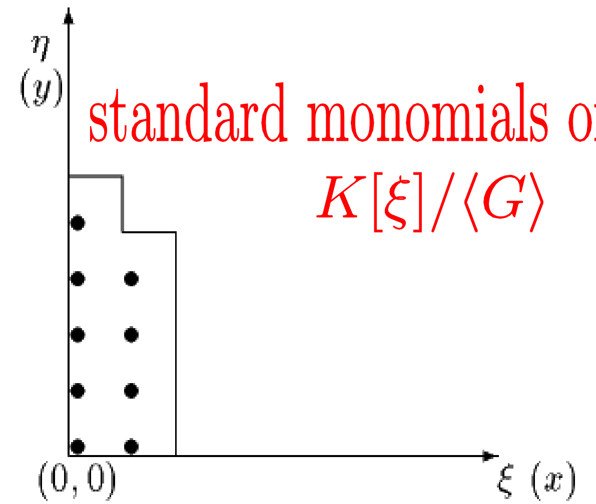
$$D_Q = \{\deg(q) \mid q \in Q\} = \{0,2,4,5,6,7,8,9,10,11,12,14,16\}$$

$\frac{\partial f_0}{\partial x} = 3x^2 + y^5, \ \frac{\partial f_0}{\partial y} = 5xy^4, \quad [x \to \xi, y \to \eta]$

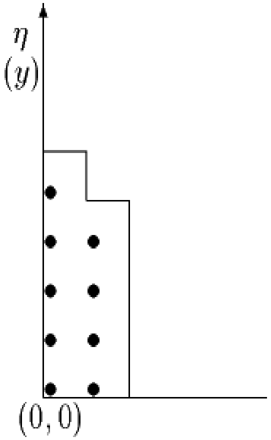
$\{\ 3\xi^2 + \eta^5, \quad 5\xi\eta^4\ \},$

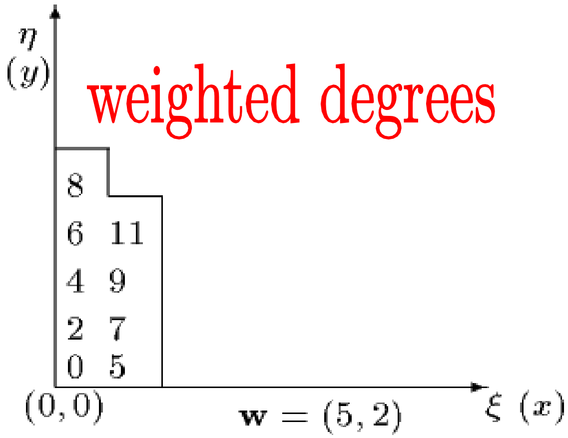
Compute the reduced Gröbner basis of $\langle \xi^2, \eta^5, \xi\eta^4 \rangle$.

The reduced Gröbner basis is $G = \{\xi^2, \eta^5, \xi\eta^4\}$.




Monomials which don't belong to $\langle G \rangle$, are elements of $Q$.




$D_Q = \{0, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 16\}$

{~~0~~, ~~2~~, ~~4~~, ~~5~~, ~~6~~, ~~7~~, ~~8~~, ~~9~~, 10, ~~11~~, 12, 14, 16}

$\{10, 12, 14, 16\}$ remained.

$10 \quad [\xi^2, \ \eta^5]$

Set $h = \xi^2 + c\eta^5$, decide $c$.

$$\frac{\partial f_0}{\partial x} * h = 3 + c = 0, \quad \frac{\partial f_0}{\partial y} * h = 0$$

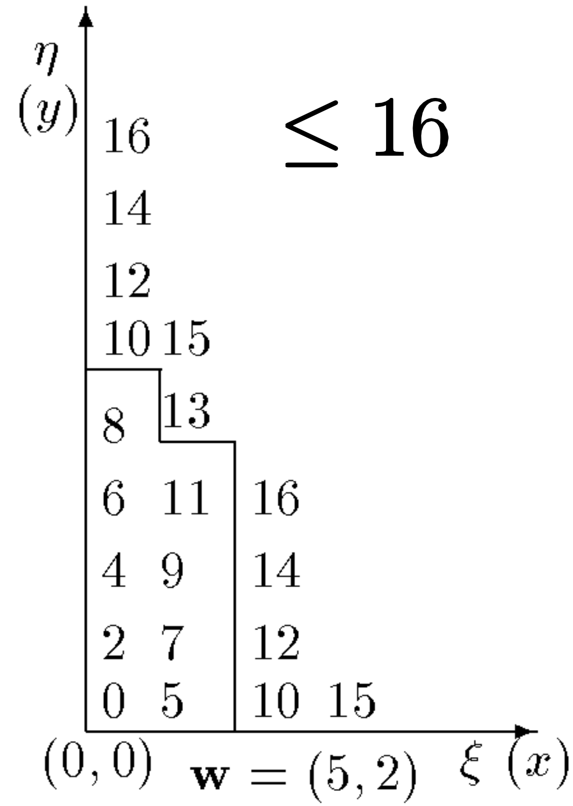
$c = -3$. Hence $\xi^2 - 3\eta^5$.

Repeat the same procedure.

$12 \quad [\xi^2\eta, \ \eta^6] \to \xi^2\eta - 3\eta^6$

$14 \quad [\xi^2\eta^2, \ \eta^7] \to \xi^2\eta^2 - 3\eta^7$

$16 \quad [\xi^2\eta^3, \ \xi^8] \to \xi^2\eta^3 - 3\eta^8$




$\leq 16$

# Semi-quasihomo. case

$f = f_0 + g \in \mathbb{C}[x]$ : **Semi-quasihomogeneous of type (d;W)**

**Theorem(The 2nd result)** Let $\{q_1, \ldots, q_k\}$ be a basis of $H_{f_0}$. For each $i = 1, \ldots, k$, there uniquely exists $r_i$ such that $\deg_{\mathbf{w}}(q_i) > \deg_{\mathbf{w}}(r_i)$ and, $h_i = q_i + r_i$ is an element of $H_f$. ($q_i$ is the head part of $h_i$ and $r_i$ is the lower part of $h_i$.) The set $\{h_1, \ldots, h_k\}$ is a basis of $H_f$.

**Fact:** A lower term of $h_i$ is not in $\{\mathrm{ht}(q_1), \ldots, \mathrm{ht}(q_k)\}$.

**Example** $f = x^3 + xy^5 + 2y^8, \quad \mathbf{w} = (5, 2)$

$f_0 = x^3 + xy^5, \quad g = 2y^8, \quad d = \deg_{\mathbf{w}}(f_0) = 15$

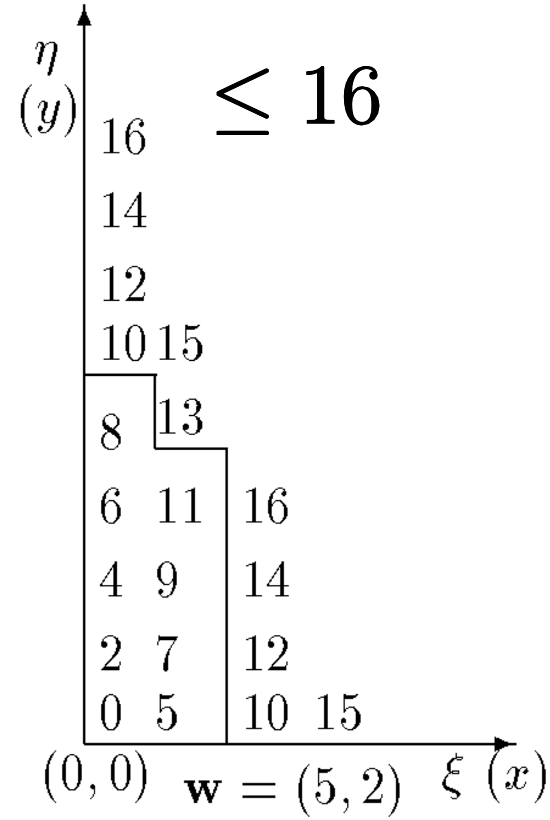
From the previous example, a basis of $H_{f_0}$ is

$$Q = \{1, \eta, \eta^2, \xi, \eta^3, \xi\eta, \eta^4, \xi\eta^2, \xi^2 - 3\eta^5, \xi\eta^3, \xi^2\eta - 3\eta^6, \xi^2\eta^2 - 3\eta^7, \xi^2\eta^3 - 3\eta^8\}$$

$$Q = \{1, \eta, \eta^2, \xi, \eta^3, \xi\eta, \eta^4, \xi\eta^2, \xi^2 - 3\eta^5, \xi\eta^3, \xi^2\eta - 3\eta^6, \xi^2\eta^2 - 3\eta^7, \xi^2\eta^3 - 3\eta^8\}$$

$$\deg_{\mathbf{w}}(\xi^2\eta^2 - 3\eta^7) = 14$$

Find lower terms, w.r.t. $\prec$, which don't belong to $\mathrm{ht}(Q)$.




candidates of the lower terms

| $\eta^5$, | $\eta^6$, | $\xi\eta^4$ |
|---|---|---|
| 10 | 12 | 13 |

Set $h = \xi^2\eta^2 - 3\eta^7 + a\eta^5 + b\eta^6 + c\xi\eta^4$,

decide $a, b, c$.

$$\frac{\partial f}{\partial x} * h = a + b\eta = 0, \quad \frac{\partial f}{\partial y} * h = 5c - 48 = 0$$

Hence, $a = b = 0, c = \dfrac{48}{5}$.

$$\boxed{\xi^2\eta^2 - 3\eta^7 + \frac{48}{5}\xi\eta^4}$$

Repeat the same procedure.

# Benchmark

**[CPU: intel Core i7-2600 3.4GHz × 2 , Memory: 4GB, OS:Windows7]**
**CPU sec.**

| Quasihomogeneous | Milnor no. | previsou alg. | Today's algo. |
|---|---|---|---|
| $f_1 := (x^4 + y^6 + x^2y^3)^2 + x^2y^9$ | **77** | 0.936 | 0.031 |
| $f_2 := (x^5 + y^7)^2 + 3y^{14}$ | **117** | 1.56 | 0.015 |
| $f_3 := (y^{13} + x^3)^2 + x^6$ | **125** | 1.669 | 0.016 |
| $f_4 := (x^4 + y^6 + x^2y^3)^3 + x^8y^6$ | **187** | 22.09 | 0.171 |
| $f_5 := (x^2z + yz^2 + y^5 + y^3z)^2 + z^5 + x^6y$ | **204** | 941.4 | 3.135 |
| $f_6 := (x^2y + z^4 + y^5)^2 + x^5 + y^5z^4$ | **252** | 1014.4 | 0.983 |
| $f_7 := (y^4 + xz^3 + x^3)^2 + y^8 + z^9$ | **280** | 1151.5 | 0.718 |
| $f_8 := (x^3y + y^7 + x^2y^3)^4 + x^{14}$ | **351** | 285.1 | 1.217 |
| $f_9 := (x^4 + y^9)^4 + 3x^{16}$ | **525** | 378.6 | 0.577 |
| $f_{10} := (x^3 + xz^2 + xy^3 + zy^3)^3$ | **800** | $1.987 \times 10^5$ | $1.619 \times 10^4$ |

| Semi-quasihomo. | Milnor no. | previous alg. | Today's algo. |
|---|---|---|---|
| $f_1 + 3x^3y^8$ | **77** | 1.263 | 0.359 |
| $f_2 + x^{10}y^5 + xy^{14}$ | **117** | 1.607 | 0.905 |
| $f_3 - 2x^3y^{20}$ | **125** | 1.544 | 0.827 |
| $f_4 + 2x^{11}y^2$ | **187** | 38.63 | 7.94 |
| $f_5 + x^3y^2z^2$ | **204** | 2023.6 | 143.2 |
| $f_6 + x^2y^3z^3$ | **252** | 1393.8 | 356.1 |
| $f_7 + xy^7$ | **280** | 1876.3 | 415.9 |
| $f_8 + x^{13}y^3$ | **351** | 893.3 | 93.85 |
| $f_9 + x^{15}y^3$ | **525** | 1244 | 287.8 |
| $f_{10} + 4x^2y^{10}z$ | **800** | $3.167 \times 10^5$ | $1.003 \times 10^5$ |

# Standard bases

**From a basis *Q* of $H_f$, a standard basis of $J = \left\langle \frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n} \right\rangle$ is easily obtained. (2009,Tajima,Nakamura, Nabeshima)**

$$H = \{\mathrm{ht}(h) | h \in Q\}, \ L : \text{a set of all lower terms of } Q,$$

$$H_i = \{\xi_i h | h \in H\}, i = 1, \ldots, n,$$

$$F = \mathrm{ReducedGr\ddot{o}bnerBasis}\left(\left(\cup_{i=1}^{n} H_i \setminus \cup_{i \neq j, i,j=1,\ldots,n} \left(H_i \cap H_j\right)\right) \setminus H\right)$$

**Theorem(2009,Tajima,Nakamura, Nabeshima)**

$$p \in Q, \ \ p \text{ has a form } p = \xi^\tau + \sum_{\tau \prec \kappa} C_{(\tau,\kappa)} \xi^\kappa.$$

$$\xi^\alpha \in F, \quad \xi^\alpha \notin L \longrightarrow x^\alpha$$

$$\xi^\alpha \in L \longrightarrow x^\alpha - \sum_{\kappa \in H} C_{(\kappa,\alpha)} x^\kappa$$

**By this mapping, *F* can be transformed into a standard basis of *J* w.r.t. the local term order.**

# Parametric Case

$$f = \underset{\uparrow \atop 27}{x^3} + \underset{\uparrow \atop 27}{y^9} + \underset{\uparrow \atop 27}{t x^2 y^3} + \underset{\uparrow \atop 30}{a y^{10}}, \quad \mathbf{w} = (9,3)$$

$t, a$ : parameters,

$$f = f_0 + g,$$
$$f_0 = x^3 + y^9 + t x^2 y^3, \quad g = a y^{10}$$

**How to compute a basis of $H_f$ ?**

**We can generalize the proposed algorithm (of non-parametric cases) to parametric cases.**

# Remarks

**In parametric cases, there is a possibility that $f_0$ does not have the isolated singularity for some values of parameters. We have to take away these values of parameters.**

**How do we compute these values of parameters ?**

**In qusihomo. case, this classification is possible by computing a comprehensive Gröbner system of the Jacobi ideal of $f_0$ .**

# Example

$$f = x^3 + y^9 + tx^2y^3 + ay^{10}, \quad \mathbf{w} = (9,3)$$

If a value of the parameter $t$ belongs to $\mathbb{V}(4t^3+27)$, then, $f$ doesn't have the isolated singularity at the origin.

A basis of $H_f$ is the following;

- If $t = 0$, then a basis of $H_f$ is
$\{1, \eta, \eta^2, \eta^3, \eta^4, \eta^5, \eta^6, \eta^7, \xi, \xi\eta, \xi\eta^2, \xi\eta^3, \xi\eta^4, \xi\eta^5, \xi\eta^6, \xi\eta^7\}$

- If $t(4t^3+27) \neq 0$, then a basis of $H_f$ is
$\{1, \eta, \eta^2, \eta^3, \eta^4, \eta^5, \eta^6, \eta^7, \xi, \xi\eta, \xi\eta^2,$
$\frac{1}{t^2}(t^2\eta\xi^3 - \frac{3}{2}t\eta^4\xi^2 + 9/4\eta^7\xi + \frac{1}{2}t^2\eta^{10} - \frac{5}{9}at^2\eta^9 + \frac{50}{81}a^2t^2\eta^8),$
$\frac{1}{t^2}(t^2\xi^3 - \frac{3}{2}t\eta^3\xi^2 + \frac{9}{4}\eta^6\xi + \frac{1}{2}t^2\eta^9 - 5/9at^2\eta^8),$
$\frac{1}{t}(t\eta^2\xi^2 - \frac{3}{2}\eta^5\xi - \frac{1}{3}t^2\eta^8), \frac{1}{t}(t\eta\xi^2 - \frac{3}{2}\eta^4\xi), \quad \frac{1}{t}(t\xi^2 - \frac{3}{2}\eta^3\xi)\}$

An algorithm for computing a basis of parametric $H_f$, has been implemented.

# Demo

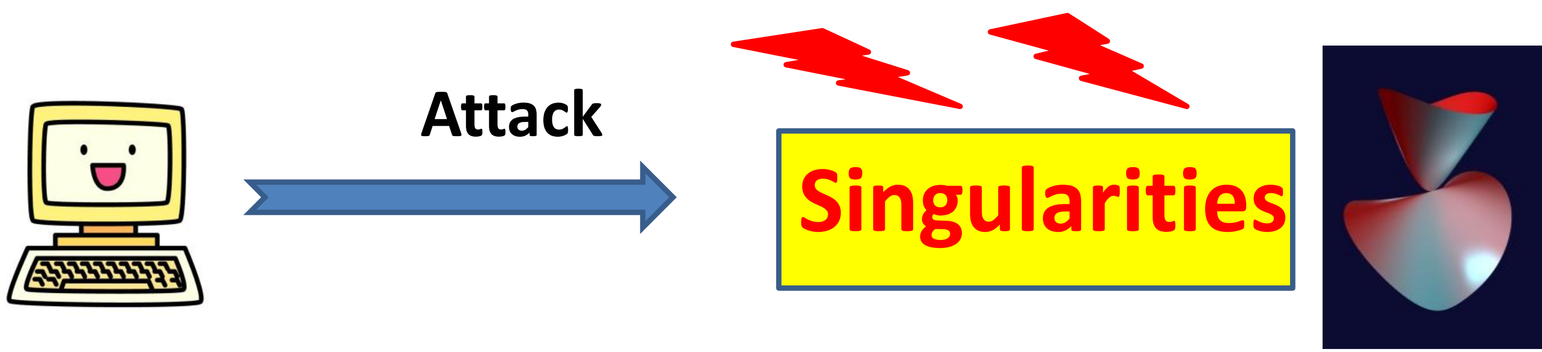
**if I have time left.**

# Conclusions

- A new algorithm for computing algebraic local cohomology classes associated with semi-quasihomo. isolated singularities, was introduced.

- The algorithm efficiently computes algebraic local cohomology classes w.r.t. a weighted term order.

- The proposed algorithm can be also extendable to handle parametric cases.




**Thank you very much.**