Solving real polynomial systems by real homotopies

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In the last few decades, the homotopy continuation method has been established in the U.S. for finding the full set of isolated solutions to a polynomial system numerically. The method involves first solving a trivial system, and then deforming these solutions along smooth paths to the solutions of the system of interest. The method has been successfully implemented and proved to be very powerful in many occasions.

While the nature setting for studying polynomial systems is the product of complex (or projective) spaces, in practice polynomial systems are almost always appeared with real coefficients, and, most importantly, only real zeros of the systems are in the wish list. One may, of course, find *all* solutions in the complex setting in the first place followed by filtering out all the real solutions. However, to deal with those systems in real spaces directly would certainly be more preferable numerically. In this talk, we will pay a special attention in solving *real* polynomial systems by *real* homotopies.

不等式机器证明的一些进展

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介绍我们最近在代数不等式机器证明方面的一些进展。这些进展主要来自于对柱形代数分解算法（cylindrical algebraic decomposition）的改进。

基于多模态特征的图像分析

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基于计算机的图像分析是图像内容自动识别与理解的基本技术途径。图像分析的实现通常是利用一个判别函数，对提取的图像特征进行计算，得到判别值。线性判别函数计算简捷，但精度较低，非线性判别精度较高，但计算复杂。图像特征一般包含多种类别，例如，颜色、形状、纹理等特征，网络图像通常与文字穿插、混合一体，文字的鲜明语义对于图像分析有着特别重要的意义，目前商业化的图像搜索软件主要利用文字进行图像检索。综合利用多模态特征进行图像分析，是目前图像识别与理解的主流研究方向。多模态特征通常维度很高，容易产生“维数诅咒”问题。因此，降维、量化技术随之得到重视，我们在特征的快速量化映射方面，提出一种基于共生概率的快速量化算法，利用已量化的特征预测相邻特征的量化值，结合树型搜索算法，取得了准确、快速的量化映射效果。针对数据分布的流形表达，提出一种组合流形正则化学习框架，类似于基函数组合，选择一组基流形，线性组合基流形来表达任意的流形分布，，流形参数和模型参数自动联合学习，任意流形分布可以得到优化的基流形与组合系数。在多模态特征的图像分类方面，提出一种基于信息瓶颈理论的多视角学习算法，利用信息瓶颈理论进行特征降维，模拟多通道的通信系统设计多视角学习模型，引入最大化间隔约束，提高低维特征的区分能力。另外证明了特征的互补性可以提高算法的鲁棒性，特征的一致性则可提高算法的泛化能力。

Quantier Elimination and Cylindrical Algebraic

Decomposition based on Regular Chains

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In this paper, we review the recent work on the study of quanti_er elimination (QE) and cylindrical algebraic decomposition (CAD) based on the theory of triangular decompositions and regular chains. The implementation of them in the RegularChains library is explained through simple examples as well as a non-trivial application.

Desingularization of linear difference operators (Extended Abstract)

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In these notes we recast and flesh out some results from [2]. In this latter paper the authors study the following related problems. Let k be a field of characterisitic zero and let $k[z]\langle E\rangle$ be the ring of linear difference operators where $Ez = (z+1)E$. Let $R \in k[z]\langle E\rangle$ be of the form

$$R = r_m E^m + \cdots + r_0, \quad \text{where } r_i \in k[z] \text{ and } r_m r_0 \neq 0. \tag{1}$$

Problem 1 (Desingularization)**.** *Let $R \in k[z]\langle E\rangle$ be a nonzero linear difference operator of the form* (1) *with r_0 having at least one root α. Decide whether there exists a nonzero operator $L \in k(z)\langle E\rangle$ of the form*

$$L = \ell_n E^n + \cdots + \ell_0, \quad \textit{where } \ell_j \in k(z) \textit{ and } \ell_n \neq 0, \tag{2}$$

such that $T = LR \in k[z]\langle E\rangle$ and its trailing coefficient is $\frac{r_0}{(z-\alpha)^\lambda}$ where λ is the multiplicity of α as a root of r_0.

Problem 2 (Extension of sequential solutions)**.** *Let R be as above. Assume that we have a sequence $s_{\alpha+1} = (s(\alpha+1), s(\alpha+2), \ldots)$ that satisfies*

$$R(s_{\alpha+1}) = \sum_{i=0}^{m} r_i(\alpha+1+n)s(\alpha+1+n+i) = 0 \quad \textit{for all } n \geq 0.$$

When can we extend $s_{\alpha+1}$ to a sequence $s_\alpha = (s(\alpha), s(\alpha+1), \ldots)$ that satisfies $R(s_\alpha) = 0$?

The key to the proofs in [2] is the analysis of a certain matrix described in Section 2 of that paper. We have replaced the consideration of this matrix with manipulations of linear difference operators in the ring $k[z]\langle E\rangle$.

1 Desingularization

In [2], the authors present an algorithm that solves Problem 1 when α is the largest integer root of r_0. On p. 125 of [1], the authors note that they have *"... implemented an algorithm that removes all singularities that can be removed, ..."*, that is, an algorithm to solve problem 1. for any α. In this section, we also show how this can be done using our methods. We also note that Abromov, Barkatou and van Hoeij present in [1] an algorithm to decide if one can give a complete desingularization of an operator (i.e., make the trailing coefficient constant) using methods different from those of [2] .

For any $\alpha \in k$ we call the set $\{\alpha + i \mid i \in \mathbb{Z}\}$ the $\mathbb{Z}$-*orbit at* α, denoted by $[\alpha]$. Let $S : k \to k$ denote the forward shift $a \mapsto a + 1$. For $\alpha_1, \alpha_2 \in [\alpha]$, we define $\alpha_1 \geq \alpha_2$ if $\alpha_1 = S^n\alpha_2$ for some $n \geq 0$. For $p \in k[z]$ and $\alpha \in k$, we denote by $[\alpha]_p$ the set $\{\beta \in \bar{k} \mid p(\beta) = 0 \text{ and } \beta \in [\alpha]\}$. Note that the set $[\alpha]_p$ is a finite set. The set of all roots of p in k can be decomposed as $\cup_i[\alpha_i]_p$, where the α_i's are in different $\mathbb{Z}$-orbits.

Definition 1. *We say that R is* desingularizable at α *if there exists $L \in k(z)\langle E\rangle$ such that $T = LR \in k[z]\langle E\rangle$ and its trailing coefficient does not have α as a root.*

Proposition 1 allows one to decide if we can desingularize at α. We do not assume that α is a maximal element of $[\alpha]_{r_0}$. We begin with the following lemma which gives a slight improvement of Lemmas 4 and 5 of [2].

Lemma 1. *Let α be a root of r_0 of multiplicity λ. Let $d \in \mathbb{N}$ be such that $r_m(\alpha + d) = 0$ and $r_m(\alpha + d + i) \neq 0$ for any positive $i \in \mathbb{Z}$. If R is desingularizable at α, then there exists $\hat{L} \in k(z)\langle E\rangle$ such that*

(i) $\hat{T} = \hat{L}R \in k[z]\langle E\rangle$,

(ii) $\mathrm{ord}(\hat{L}) \leq d$,

(iii) the trailing coefficient of $\hat{T}$ is $\frac{r_0}{(z-\alpha)^\lambda}$.

In the above Lemma, we were able to bound the order of a possible L but not the degrees of the coefficients because we could not bound μ. The next result shows that once we know the order of L is at most d, we can furthermore bound the degrees of the coefficients. We could have combined these proofs but feel separating them may help in understanding the basic ideas involved.

Before we can state the main result we define some notation. For $\beta \in [\alpha]_{r_0}$, let $\lambda_\beta =$ the multiplicity of β as a root of r_0 and let

$$\begin{aligned} \rho &= \max\{\lambda_\beta \mid \beta \in [\alpha]_{r_0} \text{ and } \beta > \alpha\} \\ \lambda &= \lambda_\alpha \end{aligned}$$

Note that if α is the maximum element of $[\alpha]_{r_0}$ then $\rho = 0$. The following allows us to decide if we can desingularize R at α.

Proposition 1. *Let α be a root of r_0 and λ and ρ as above. Let $d \in \mathbb{N}$ be such that $r_m(\alpha+d) = 0$ and $r_m(\alpha+d+i) \neq 0$ for any positive $i \in \mathbb{Z}$. If R is desingularizable at α, then there exists $\hat{L} \in k(z)\langle E\rangle$ such that*

(i) $\hat{T} = \hat{L}R \in k[z]\langle E\rangle$,

(ii) $\mathrm{ord}(\hat{L}) \leq d$,

(iii) the coefficients $\hat{\ell}_i$ of $\hat{L}$ are of the form

$$\hat{\ell}_i = \frac{n_i(z)}{(z-\alpha)^{\lambda+d\rho}}$$

$n_i \in k[z], \deg_z(n_i(z)) \leq \lambda + d\rho - 1.$

(iv) the trailing coefficient of $\hat{T}$ is $\frac{r_0}{(z-\alpha)^{\lambda}}$.

2 Extension of sequential solutions

For any $\beta \in k$, let s_β denote the right-sided sequence

$$\{s(\beta), s(\beta+1), s(\beta+2), \ldots\}, \quad \text{where } s(\beta+i) \in k \text{ for all } i \in \mathbb{N}. \tag{3}$$

If s_β is a right-sided sequence and $i \in \mathbb{N}$, we denote by $s_{\beta+i}$ the right-sided sequence $(s(\beta+i), s(\beta+i+1), \ldots)$. We say s_α is an *extension* of s_β if $s_\beta = s_{\alpha+i}$ for some $i \in \mathbb{N}$. For $P = \sum_{i=0}^{d} p_i E^i \in k[z]\langle E\rangle$, we say that s_β is a *sequential solution* of a linear recurrence $P(Y) = 0$ if

$$\sum_{i=0}^{d} p_i(\beta+n)s(\beta+n+i) = 0 \quad \text{for all } n \geq 0.$$

We will use the notation $P(s_\beta) = 0$ to denote that s_β is a sequential solution of $P(Y) = 0$.

Let s_β be a sequential solution of a linear recurrence $P(Y) = 0$, $P = \sum_{i=0}^{d} p_i E^i \in k[z]\langle E\rangle$. We wish to give criteria which will allow one to extend s_β to a sequential solution s_α of $P(Y) = 0$ where $\alpha = \beta - n$ for some $n \in \mathbb{N}$. We start with the following technical lemma.

Lemma 2. *Let $\alpha, \beta \in k$ such that $\beta - \alpha \in \mathbb{N}^+$ and let j be a positive integer, $0 < j \leq \beta - \alpha$.*

1. *Let $U_1, \ldots, U_j$ be elements of $k[z]\langle E\rangle$ such that for each $i, i = 1, \ldots, j$, $u_{i,0}(\beta - i) \neq 0$ where $u_{i,0}$ is the coefficient of E^0 in U_i. Let $U \in k[z]\langle E\rangle$. Then there exist polynomials $a, a_0, \ldots, a_{j-1} \in k[z]$ with $a(\beta - j) \neq 0$ such that*

$$aU - \sum_{i=0}^{j-1} a_i E^i U_{j-i} \;\; = \;\; \tilde{u}_N E^N + \ldots + \tilde{u}_j E^j$$

for some $\tilde{u}_i \in k[z]$.

2. Let

$$\begin{aligned} R &= r_m E^m + \ldots + r_0 \text{ and} \\ V &= v_{m+n} E^{m+n} + \ldots + v_j E^j \in k[z]\langle E\rangle \end{aligned}$$

where $n \geq 0$ and $r_m r_0 \neq 0$. Assume that $r_0(\beta + i) \neq 0$ for all $i \in \mathbb{N}$. Then there exist $b, b_0, \ldots b_{n-j} \in k[z]$ with $b(\beta - j) \neq 0$ such that

$$bV - \sum_{i=0}^{n-j} b_i E^{i+j} R \;=\; \tilde{v}_{m+n} E^{m+n} + \ldots + \tilde{v}_{n+1} E^{n+1}$$

for some $\tilde{v}_i \in k[z]$.

We can now prove the following (corresponding to Theorem 2 of [2]).

Proposition 2. *Let $R, T_1, \ldots, T_s \in k[z]\langle E\rangle, \operatorname{ord}(R) = m > 0$. Let $r_0, t_{1,0}, \ldots, t_{s,0}$ be the coefficients of E^0 in $R, T_1, \ldots, T_s$ respectively and let $\alpha, \beta \in k$ such that $\beta - \alpha \in \mathbb{N}^+$. Assume that*

(i) For each $i \in \mathbb{N}, 0 < i \leq \beta - \alpha$, at least one of the values of $r_0(\beta - i), t_{1,0}(\beta - i), \ldots, t_{s,0}(\beta - i)$ is not 0.

(ii) For all $i \in \mathbb{N}$ each of the values $r_0(\beta + i)$ is not zero.

(iii) Each T_j is right divisible by R.

If s_β is a sequential solution of $R(Y) = T_1(Y) = \ldots = T_s(Y) = 0$, then s_β can be extended uniquely to a sequential solution s_α of the same equations.

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一类交错和的一个利用有限嵌套和表示的简洁公式

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摘 要: 本文基于 WZ 理论框架给出了一类形如 $\sum_{k=0}^{n}\binom{n}{k}\frac{(-1)^k}{(k+\xi)^p}$ (其中 $\xi, p \in N$ 均为一般参数)的交错和的一个利用有限嵌套和来表示的简洁公式:

定理. 设 $\xi, p \in N$, 则有

$$\sum_{k=0}^{n}\binom{n}{k}\frac{(-1)^k}{(k+\xi)^p}=\binom{n+\xi}{n}^{-1}\left[\frac{1}{\xi^p}+\sum_{k=1}^{p-1}\left(\sum_{j_1=1}^{n}\frac{1}{j_1+\xi}\sum_{j_2=1}^{j_1}\frac{1}{j_2+\xi}\cdots\sum_{j_k=1}^{j_{k-1}}\frac{1}{j_k+\xi}\right)\frac{1}{\xi^{p-k}}\right],$$

这里我们约定 $\sum_{k=1}^{0}b_k=0$.

实际上, 定理的结果对 $\xi \in C\setminus\{0,-1,-2,\cdots\}$ 也是成立的, 由此结果及下述公式

$$x(x+1)\cdots(x+n-1)=\sum_{j=0}^{n}\begin{bmatrix}n\\ j\end{bmatrix}x^j,$$

其中 $\begin{bmatrix}n\\ j\end{bmatrix}$ 为第一类 Stirling 数, 可证明: 对 $\forall p \in N$, 均有

$$\sum_{k=1}^{n}\binom{n}{k}\frac{(-1)^{k-1}}{k^p}=\sum_{j_1=1}^{n}\frac{1}{j_1}\sum_{j_2=1}^{j_1}\frac{1}{j_2}\cdots\sum_{j_p=1}^{j_{p-1}}\frac{1}{j_p}\left(=\sum_{1\le j_1\le j_2\le\cdots\le j_p\le n}\frac{1}{j_1 j_2\cdots j_p}\right),$$

这是 Gould 的已知的结果, 在证明上述结果的同时, 我们还获得了下述公式: 对 $\forall k=1,2,\cdots$, 均有

$$\frac{1}{n!}\sum_{j=k}^{n}\begin{bmatrix}n\\ j\end{bmatrix}\binom{j}{k}=\sum_{j_1=1}^{n}\frac{1}{j_1}\sum_{j_2=1}^{j_1}\frac{1}{j_2}\cdots\sum_{j_k=1}^{j_{k-1}}\frac{1}{j_k}.$$

EFFICIENTLY COUNTING AFFINE ROOTS OF MIXED TRIGONOMETRIC POLYNOMIAL SYSTEMS

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Abstract. Estimating the root count, which is the total number of geometrically isolated solutions, of a polynomial system is not only a fundamental study theme in algebraic geometry but also an important subproblem of homotopy continuation methods for solving polynomial systems. For the mixed trigonometric polynomial systems, which are more general than polynomial systems and rather frequently occur in many applications, the classical Bézout number and the multihomogeneous Bézout number are the best known upper bounds on the root count. However, for the deficient mixed trigonometric polynomial systems arising in practice, all these upper bounds are far greater than the actual root count. The BKK bound is known as the most accurate upper bound on the root count of polynomial systems. However, the extension of the definition of the BKK bound allowing it to treat mixed trigonometric polynomial systems is very difficult due to the existence of sine and cosine functions. In this paper, two new upper bounds on the root count of a mixed trigonometric polynomial system are defined and the corresponding efficient algorithms for calculating them are also presented. Numerical tests are also given to show the accuracy of these two definitions, and numerically prove they can provide tighter upper bounds on the root count of a mixed trigonometric polynomial system than the existent upper bounds, and also we compare the computational time for calculating these two upper bounds.

$SL(4)$与SO(3,3)之间的一个对应关系及其在直线几何中应用

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摘要：关于直线几何学，我们的研究进展如下：

一、给出了$SL(4)$到SO(3,3)的一个双重覆盖同态映射，并给出了SO(3,3)中元素在此映射下的原像.

用四维齐次坐标表示三维射影空间$\mathbb{P}^3$中的一个点，这样$\mathbb{P}^3$上一个非退化线性变换ϕ将$\mathbb{P}^3$中一个点变成（另）一个点，这个非退化线性变换关于$\mathbb{P}^3$的一个基$\mathrm{e}_0,\mathrm{e}_1,\mathrm{e}_2,\mathrm{e}_3$的矩阵$A$也非退化，不妨设其行列式为1，即设其为 $SL(4)$中的元素，这里$SL(4)=\{A\in\mathbb{R}^{4\times4}|\det(A)=1\}$. 该线性变换还把三维空间中直线变成直线，当直线用其Plüker坐标表示时，线性变换ϕ就诱导出六维线性空间上的一个线性变换ψ，ψ在基$\mathrm{e}_{01},\mathrm{e}_{02},\mathrm{e}_{03},\mathrm{e}_{23},\mathrm{e}_{31},\mathrm{e}_{12}$下的矩阵为$B$. 可以证明$B\in SO(3,3)$，这里$SO(3,3)=\{M\in\mathbb{R}^{6\times6}|M^{\mathrm{T}}J_6M=J_6,\det(M)=1\}$，其中$J_6=\begin{pmatrix}O & I_3\\ I_3 & O\end{pmatrix}$，$I_3$是三阶单位矩阵. 这就建立了$SL(4)$与SO(3,3)之间一个二重覆盖映射$f: SL(4)\to SO(3,3); A\mapsto B$，具体写出来就是

$$\begin{pmatrix}a_{00} & a_{01} & a_{02} & a_{03}\\ a_{10} & a_{11} & a_{12} & a_{13}\\ a_{20} & a_{21} & a_{22} & a_{23}\\ a_{30} & a_{31} & a_{32} & a_{33}\end{pmatrix}\mapsto\begin{pmatrix}A_{01}^{01} & A_{02}^{01} & A_{03}^{01} & A_{23}^{01} & A_{31}^{01} & A_{12}^{01}\\ A_{01}^{02} & A_{02}^{02} & A_{03}^{02} & A_{23}^{02} & A_{31}^{02} & A_{12}^{02}\\ A_{01}^{03} & A_{02}^{03} & A_{03}^{03} & A_{23}^{03} & A_{31}^{03} & A_{12}^{03}\\ A_{01}^{23} & A_{02}^{23} & A_{03}^{23} & A_{23}^{23} & A_{31}^{23} & A_{12}^{23}\\ A_{01}^{31} & A_{02}^{31} & A_{03}^{31} & A_{23}^{31} & A_{31}^{31} & A_{12}^{31}\\ A_{01}^{12} & A_{02}^{12} & A_{03}^{12} & A_{23}^{12} & A_{31}^{12} & A_{12}^{12}\end{pmatrix},$$

这里A_{31}^{02}表示取矩阵A的第1、3行，第4、2列交叉处元素按顺序组成的二阶行列式$\begin{vmatrix}a_{03} & a_{01}\\ a_{23} & a_{21}\end{vmatrix}$，其余类同. 还可以证明这一映射保持矩阵的乘法运算，即它是一个同态.

反过来，对于任意$B=\begin{pmatrix}b_{11} & b_{12} & b_{13} & b_{11'} & b_{12'} & b_{13'}\\ b_{21} & b_{22} & b_{23} & b_{21'} & b_{22'} & b_{23'}\\ b_{31} & b_{32} & b_{33} & b_{31'} & b_{32'} & b_{33'}\\ b_{1'1} & b_{1'2} & b_{1'3} & b_{1'1'} & b_{1'2'} & b_{1'3'}\\ b_{2'1} & b_{2'2} & b_{2'3} & b_{2'1'} & b_{2'2'} & b_{2'3'}\\ b_{3'1} & b_{3'2} & b_{3'3} & b_{3'1'} & b_{3'2'} & b_{3'3'}\end{pmatrix}\in SO(3,3)$，我们找到了它在映射$f$

下的原像 $A=(a_{ij}),\quad i,j=0,1,2,3$，易知 A 的第一行元素不全为零，计算得 $a_{00}=\pm\sqrt{B_{123}^{123}}$，这里 B_{123}^{123} 表示取矩阵 B 的第 1、2、3 行, 第 1、2、3 列交叉处元素所得的 B 的三阶子式. 不妨设 $a_{00}\neq 0$，则

$$a_{00}A=\frac{1}{2}\begin{pmatrix} 2B_{123}^{123} & 2B_{212'}^{123} & 2B_{323'}^{123} & 2B_{131'}^{123} \\ 2B_{123}^{212'} & B_{212'}^{212'}+B_{313'}^{212'}+b_{11} & B_{323'}^{212'}+B_{121'}^{212'}+b_{12} & B_{131'}^{212'}+B_{232'}^{212'}+b_{13} \\ 2B_{123}^{323'} & B_{212'}^{323'}+B_{313'}^{323'}+b_{21} & B_{323'}^{323'}+B_{121'}^{323'}+b_{22} & B_{131'}^{323'}+B_{232'}^{323'}+b_{23} \\ 2B_{123}^{131'} & B_{212'}^{131'}+B_{313'}^{131'}+b_{31} & B_{323'}^{131'}+B_{121'}^{131'}+b_{32} & B_{131'}^{131'}+B_{232'}^{131'}+b_{33} \end{pmatrix}.$$

这样原像 $A=(a_{ij}),\quad i,j=0,1,2,3$ 就求出来了. 在计算过程中如果 $B_{123}^{123}<0$，则按同样方法计算 $-B$ 的原像即可. 实际上映射 f 不是满射，f 的像集只是 $SO(3,3)$ 中元素的一半，另一半元素只与像集中相应元素相差一个符号.

二、给出了 $\mathbb{R}^{3,3}$ 中特殊正交变换——镜面反射在某基下的矩阵，并给出两个此类矩阵之积在上面二重覆盖下的原像，进一步给出了 $SL(4)$ 中矩阵 A 可以写成两个反对称矩阵之积的充分必要条件.

对于六维线性空间中镜面反射 $Ad_u^*(x)=x-\dfrac{2x\cdot u}{u\cdot u}u$，其中 $u=\lambda_1e_{01}+\lambda_2e_{02}+\lambda_3e_{03}+\lambda_1'e_{01}'+\lambda_2'e_{02}'+\lambda_3'e_{03}'$，且 $u\cdot u=2(\lambda_1\lambda_1'+\lambda_2\lambda_2'+\lambda_3\lambda_3')\neq 0$，这里分别以 e_{01}',e_{02}',e_{03}' 代替了 e_{23},e_{31},e_{12}. 令 $\lambda=\lambda_1\lambda_1'+\lambda_2\lambda_2'+\lambda_3\lambda_3'$，计算得 Ad_u^* 在基 $e_{01},e_{02},e_{03},\ e_{01}',e_{02}',e_{03}'$ 下的矩阵为

$$M_u=\frac{1}{\lambda}\begin{pmatrix} \lambda_2\lambda_2'+\lambda_3\lambda_3' & -\lambda_2'\lambda_1 & -\lambda_3'\lambda_1 & -\lambda_1\lambda_1 & -\lambda_2\lambda_1 & -\lambda_3\lambda_1 \\ -\lambda_1'\lambda_2 & \lambda_1\lambda_1'+\lambda_3\lambda_3' & -\lambda_3'\lambda_2 & -\lambda_1\lambda_2 & -\lambda_2\lambda_2 & -\lambda_3\lambda_2 \\ -\lambda_1'\lambda_3 & -\lambda_2'\lambda_3 & \lambda_1\lambda_1'+\lambda_2\lambda_2' & -\lambda_1\lambda_3 & -\lambda_2\lambda_3 & -\lambda_3\lambda_3 \\ -\lambda_1'\lambda_1' & -\lambda_2'\lambda_1' & -\lambda_3'\lambda_1' & \lambda_2\lambda_2'+\lambda_3\lambda_3' & -\lambda_2\lambda_1' & -\lambda_3\lambda_1' \\ -\lambda_1'\lambda_2' & -\lambda_2'\lambda_2' & -\lambda_3'\lambda_2' & -\lambda_1\lambda_2' & \lambda_1\lambda_1'+\lambda_3\lambda_3' & -\lambda_3\lambda_2' \\ -\lambda_1'\lambda_3' & -\lambda_2'\lambda_3' & -\lambda_3'\lambda_3' & -\lambda_1\lambda_3' & -\lambda_2\lambda_3' & \lambda_1\lambda_1'+\lambda_2\lambda_2' \end{pmatrix}.$$

由于 $\det(M_u)=-1$，故 $M_u\notin SO(3,3)$，但是 M_uJ_6 与 J_6M_u 均在 $SO(3,3)$ 中，且当 $\lambda<0$ 时，M_uJ_6 与 J_6M_u 在 f 下的原像分别为

$$A_u^l = \pm\frac{1}{\sqrt{|\lambda|}}\begin{pmatrix} 0 & \lambda_1 & \lambda_2 & \lambda_3 \\ -\lambda_1 & 0 & \lambda_3' & -\lambda_2' \\ -\lambda_2 & -\lambda_3' & 0 & \lambda_1' \\ -\lambda_3 & \lambda_2' & -\lambda_1' & 0 \end{pmatrix} \text{与} A_u^r = \pm\frac{1}{\sqrt{|\lambda|}}\begin{pmatrix} 0 & \lambda_1' & \lambda_2' & \lambda_3' \\ -\lambda_1' & 0 & \lambda_3 & -\lambda_2 \\ -\lambda_2' & -\lambda_3 & 0 & \lambda_1 \\ -\lambda_3' & \lambda_2 & -\lambda_1 & 0 \end{pmatrix};$$

当 $\lambda > 0$ 时，$-M_u J_6$ 与 $-J_6 M_u$ 在 f 下的原像分别为 A_u^l 与 A_u^r .

对于 $v = \mu_1 e_{01} + \mu_2 e_{02} + \mu_3 e_{03} + \mu_1' e_{01}' + \mu_2' e_{02}' + \mu_3' e_{03}'$，可得线性变换 Ad_v^* 在基 $e_{01}, e_{02}, e_{03}, e_{01}', e_{02}', e_{03}'$ 下的矩阵为

$$M_v = \frac{1}{\mu}\begin{pmatrix} \mu_2\mu_2' + \mu_3\mu_3' & -\mu_2'\mu_1 & -\mu_3'\mu_1 & -\mu_1\mu_1 & -\mu_2\mu_1 & -\mu_3\mu_1 \\ -\mu_1'\mu_2 & \mu_1\mu_1' + \mu_3\mu_3' & -\mu_3'\mu_2 & -\mu_1\mu_2 & -\mu_2\mu_2 & -\mu_3\mu_2 \\ -\mu_1'\mu_3 & -\mu_2'\mu_3 & \mu_1\mu_1' + \mu_2\mu_2' & -\mu_1\mu_3 & -\mu_2\mu_3 & -\mu_3\mu_3 \\ -\mu_1'\mu_1' & -\mu_2'\mu_1' & -\mu_3'\mu_1' & \mu_2\mu_2' + \mu_3\mu_3' & -\mu_2\mu_1' & -\mu_3\mu_1' \\ -\mu_1'\mu_2' & -\mu_2'\mu_2' & -\mu_3'\mu_2' & -\mu_1\mu_2' & \mu_1\mu_1' + \mu_3\mu_3' & -\mu_3\mu_2' \\ -\mu_1'\mu_3' & -\mu_2'\mu_3' & -\mu_3'\mu_3' & -\mu_1\mu_3' & -\mu_2\mu_3' & \mu_1\mu_1' + \mu_2\mu_2' \end{pmatrix},$$

其中 $\mu = \mu_1\mu_1' + \mu_2\mu_2' + \mu_3\mu_3'$.

线性变换 Ad_u^* 与 Ad_v^* 的乘积 $Ad_u^* Ad_v^*$ 在基 $e_{01}, e_{02}, e_{03}, e_{01}', e_{02}', e_{03}'$ 下的矩阵为 $M_u M_v = M_u J_6 J_6 M_v$，该矩阵在 f 下的原像为

$$A = \pm\frac{1}{\sqrt{|\lambda\mu|}}\begin{pmatrix} -\lambda_1\mu_1' - \lambda_2\mu_2' - \lambda_3\mu_3' & -\lambda_2\mu_3 + \lambda_3\mu_2 & -\lambda_3\mu_1 + \lambda_1\mu_3 & -\lambda_1\mu_2 + \lambda_2\mu_1 \\ \lambda_2'\mu_3' - \lambda_3'\mu_2' & -\lambda_1\mu_1' - \lambda_2'\mu_2 - \lambda_3'\mu_3 & -\lambda_1\mu_2' + \lambda_2'\mu_1 & -\lambda_1\mu_3' + \lambda_3'\mu_1 \\ \lambda_3'\mu_1' - \lambda_1'\mu_3' & \lambda_1'\mu_2 - \lambda_2\mu_1' & -\lambda_1'\mu_1 - \lambda_2\mu_2' - \lambda_3'\mu_3 & -\lambda_2\mu_3' + \lambda_3'\mu_2 \\ \lambda_1'\mu_2' - \lambda_2'\mu_1' & \lambda_1'\mu_3 - \lambda_2\mu_1' & \lambda_2'\mu_3 - \lambda_3\mu_2' & -\lambda_1'\mu_1 - \lambda_2'\mu_2 - \lambda_3\mu_3' \end{pmatrix}.$$

进一步地，证明了 $SL(4)$ 中矩阵 A 可以写成两个反对称矩阵之积的充分必要条件是矩阵 $(f(A) - I_6)$（或 $(f(A) + I_6)$）的秩为 2.

SPECULAR SURFACE MEASUREMENT USING B-SPLINE CURVED SURFACE FOR IMAGING SIMULATION

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Our work focuses on the smooth and continuous convex specular surfaces measurement using a camera and a LCD display. We firstly analyze the geometric characteristics of a specular surface and prove that the 3D shape of the specular surface can be measured by one camera. A uniform bicubic B-spline surface is used to simulate the imaging process in the measurement due to its similarity of geometric characteristics from a specular surface. We obtain the 3D shape of the specular object by minimizing sum of the errors between the real points on the display and the intersecting points which produced by backward ray-tracing reflection rays. A 3-step phase-shifting method is applied to encode the display for corresponding the pixels on the display and image plane. Our experiments on real specular surface show that the method is simple and obtain high accuracy measurement result.

On the "normal" error formula for bivariate ideal interpolation

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Abstract. In this paper we investigate an error formula for bivariate ideal interpolation. We shall call it the "normal" error formula which derives from the "good" error formula raised by Carl de Boor. We prove that a lexicographic order reduced Gröbner basis admits such an error formula. In 2010, Boris Shekhtman proves the ideal projector P_* defined by $\ker P_* = \langle x^2 - y, xy, y^2 \rangle$ does not have a "good" error formula. As an example, we will show such a P_* has a "normal" error formula.

The Breadth-one D-invariant Polynomial Subspace

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Abstract. We demonstrate the equivalence of two classes of D-invariant polynomial subspaces introduced in [8] and [9], i.e., these two classes of subspaces are different representations of the breadth-one D-invariant subspace. Moreover, we solve the discrete approximation problem in ideal interpolation for the breadth-one D-invariant subspace. Namely, we find the points, such that the limiting space of the evaluation functionals at these points is the functional space induced by the given D-invariant subspace, as the evaluation points all coalesce at one point.

POLYNOMIAL HOMOTOPY METHOD FOR SOLVING SPARSE INTERPOLATION PROBLEM

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In this paper, the solutions of the polynomial system arising from sparse interpolation problems are studied. Exploiting the special structure of the polynomial system, it is proved that: for generic data, all its solutions belong to one equivalence class if the sampling are equally spaced. For some special unequally spaced sampling, we give the upper-bound on the number of solutions of the corresponding polynomial system. Based on the coefficient parameter homotopy method, an efficient algorithm is proposed. Unlike some existing algorithms, the proposed algorithm does not require the assumption of equally spaced sampling or positive weighted coefficients, and it is globally convergent. Preliminary numerical tests show that the new method is promising.

On the Equivalence of Multivariate Polynomial Matrices

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The equivalence of system is an important concept in multidimensional (nD) system, which is closely related to equivalence of multivariate polynomial matrices. This paper mainly investigates the equivalence of some nD polynomial matrices, and obtains several new results on the reduction by equivalence of some given nD polynomial matrices to simpler forms:

Theorem 1. Let $F(z) \in R^{l\times l}[z]$ with $\det F(z) = \prod_{i=1}^{k}(z_1 - f_i(z_2, \cdots, z_n))^{q_i}$. If $\det F(z)$ and all $(l-1)\times(l-1)$ minors of $F(z)$ have no common zeros ($F(z)$ has property DM), then $F(z)$ is equivalent to

$$V(z)P_{11}(z)U_{11}(z)\cdots P_{1q_1}(z)U_{1q_1}(z)P_{21}(z)U_{21}(z)\cdots P_{2q_2}(z)U_{2q_2}(z)\cdots P_{kq_k}(z)U_{kq_k}(z)$$

where

$$P_{ij_i}(z) = \begin{pmatrix} I_{l-1} & 0_{l-1,1} \\ 0_{1,l-1} & z_1 - f_i(z_2, \cdots, z_n) \end{pmatrix}$$

$V(z), U_{i,j_i}(z) \in SL_l(R[z]), j_i = 1, 2, \cdots, q_i, i = 1, 2, \cdots k.$

Theorem 2. Let $F(z) \in R^{2\times 2}[z]$ with $detF(z) = (z_1 - f(z_2, \cdots, z_n))^q$. If $F(z)$ has the property DM, then $F(z)$ is equivalent to the matrix

$$Q(z) = \begin{pmatrix} 1 & 0 \\ 0 & (z_1 - f(z_2, \cdots, z_n))^q \end{pmatrix}$$

Theorem 3. Let $F(z) \in R^{l\times l}[z]$ with $\det F(z) = (z_1 - f(z_2, \cdots, z_n))^q$. $F(z)$ has the property DM if and only if $F(z)$ is equivalent to a matrix $Q(z)$ with

$$Q(z) = \begin{pmatrix} I_{l-1} & 0_{l-1,1} \\ 0_{1,l-1} & (z_1 - f(z_2, \cdots, z_n))^q \end{pmatrix}$$

Efficient Slicing of Face-vertex Triangle Mesh for Addictive Manufacturing

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Abstract

This paper present an efficient slicing procedure of face-vertex triangle mesh for addictive manufacturing. The proposed algorithm introduces the use of connectivity representation of face-vertex triangle mesh, including incidence, adjacency, and ordering, in the slicing of triangle mesh. Secondly, based on the triangle mesh interval tree data structure, efficient intersection query method of triangle mesh and slicing plane is established, and the method can find all the triangles in a large scale triangle mesh that intersect the given slicing plane in $O(\log n + k)$ time, where k is the number of retrieved triangles. Finally, an efficient slicing algorithm for addictive manufacturing, which is based on the connectivity representation of face-vertex triangle mesh and triangle-plane intersection query method, is proposed. The intersection calculation of plane and triangle mesh is discussed in four different situations. Thanks to the use of triangle mesh connectivity representation, the triangle mesh slicing algorithm proves to be very efficient. Several experimental results are studied to verify the robustness and performance of the proposed algorithm.

Keywords: Addictive manufacturing; Triangle mesh; Machining path

The generalized Serre Problem over K-Hermite Rings

Jinwang Liu Dongmei Li Mingsheng Wang

A commutative ring R is called K-Hermite if, for any rectangular matrix $B \in M_{m\times n}(R)(m \le n)$, there exists $Q \in GL_n(R)$ such that BQ is lower triangular. This paper investigates the completion and the zero prime factorization for matrices over a K-Hermite ring. We have generalized Kaplansky's related results, solved similar Lin-Bose problem and generalized Serre problem over a K-Hermite ring.

Theorem 1 Let R be a d^*-Hermite ring, $F \in M_{l\times m}(R)$ $(l \le m \le d+2)$ be any ZLP matrix. Then F can be completed to a square $m \times m$ invertible matrix A over R.

Theorem 2 Let R be a K-Hermite ring. For a row vector $(a_1, a_2, \ldots, a_n)$ of R, and any maximal common divisor d of $a_1, a_2, \ldots, a_n$. Then $(a_1, a_2, \ldots, a_n)$ can be completed to a square matrix

$$\begin{pmatrix} a_1, a_2, \cdots, a_n \\ N \end{pmatrix}$$

whose determinant is d. Furthermore, the $(n-1)\times n$ matrix N may be chosen to be itself completed to a matrix in $GL_n(R)$.

Theorem 3 Let R be a K-Hermite ring, $A \in M_{l\times n}(R)(l \le n)$ be full row rank. Then for any maximal common divisor d of all $l \times l$ minors of A, we have $A = D \cdot A_1$, where $D \in M_{l\times l}(R), A_1 \in M_{l\times n}(R), \det D = d, A_1$ is ZLP.

Theorem 4 Let R be a K-Hermite ring, $A \in M_{l\times n}(R)$, with $l < n$. Then A can be completed to a square matrix $\begin{pmatrix} A \\ N \end{pmatrix}$ whose determinant is a maximal common divisor of all $l \times l$ minors of A. Furthermore, the $(n-l)\times n$ matrix N may be completed to a matrix in $GL_n(R)$.

Theorem 5 Let R be a Bézout ring in which every zero-divisor lies in Rad(R), $A \in M_{l\times n}(R)(l < n)$, and d be any maximal common divisor of all $l \times l$ minors of A. Then, A can be completed to a square matrix $\begin{pmatrix} A \\ N \end{pmatrix}$ whose determinant is d. Furthermore, the $(n-l)\times n$ matrix N can be completed to a matrix in $GL_n(R)$.

An Improved Laplace Decomposition Method For Solving Nonlinear Differential System

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In this paper, a flaw of the Laplace decomposition method is investigated. Based on it a modified Laplace decomposition method is presented, the new algorithm can be applied to a wider range of partial differential equations. We further compare advantages and disadvantages of these two methods. It can be seen that they have different strong points for different kinds of partial differential equations, while they will be reduced to the same algorithm for ordinary differential equations.

基于STL 文件的3D 打印分层算法研究及实现

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本文提出了一种能有效保留模型特征的改进的自适应分层算法。基于模型是否可打印，本文又提出了能有效检测STL 文件和CLI 文件各种错误的方法。基于Qt 和OpenGL，用C++开发了一款分层软件，该软件主要用于实现上面的分层算法和检错功能，并提供了基于3D 打印的一些其他基本功能。

非线性系统极小二范数解的可信误差界 *

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摘要: 本文定义欠定非线性系统的极小二范数解为解向量二范数的极小值点, 超定非线性系统的极小二范数解为残差向量二范数的极小值点. 本文将方形系统单根的可信验证方法和对称正定矩阵的可信验证方法相结合, 给出计算雅可比矩阵为列满秩的非线性系统的极小二范数解的可信误差界的算法. 给定非线性系统的一个近似解, 若该算法成功输出区间向量, 则在该区间向量内必定存在该系统的一个极小二范数解.

关键词: 欠定系统,超定系统, 极小二范数解, 可信验证.

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Verified error bounds for minimum 2-norm solutions of nonlinear systems

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Abstract: In this paper, we define the minimum 2-norm solution of underdetermined nonlinear system as the minimum point of the 2-norm of solution vectors, and the minimum 2-norm solution of overdetermined nonlinear system as the minimum point of the 2-norm of residual vectors. Combining with the verification for the simple solution of square system and the verification for symmetric positive definite matrix, we present an algorithm for verifying the minimum 2-norm solution of underdetermined and overdetermined nonlinear systems with full rank Jacobian matrix. Given an approximate solution of a nonlinear system, if this algorithm successfully outputs an interval vector, then there exists a minimum 2-norm solution in the interval vector.

Key words: Underdetermined system, Overdetermined system, Minimum 2-norm solution, Verification.

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A Symbolic Computation Approach to Designing Parametric Controller for Hopf Bifurcation System

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***Abstract* - In this paper, we use the Lorenz system as an illustrative example to present a method for Hopf bifurcation control of nonlinear dynamical systems. The parametric controller designed by this method has a generic explicit formula and it is derived for controlling bifurcations using nonlinear state feedback. The controller under which the equilibria of the original system remain unchanged is consisted of vector field of the polynomial system. Symbolic computation is applied to obtain the stable constraints, for the system added parametric controller. This approach employs Cylindrical Algebraic Decomposition (CAD) to find stability parameter space of the controller from the inequities of stable constraints. The simulation results are presented to confirm the analytical predictions.**

Multi-boundary Shape Retrieval Based on a New Class of Moment Functions

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Abstract *Based on a class of complete orthogonal function system, V-system, this paper proposes a new kind of moment functions (called V-moment functions), and applies them to the shape retrieval. The V-moments are orthogonal, and involve only simple linear operations. The V-moments can be used to extract image features accurately, and the original image can be reconstructed with only a small amount of them. The V-moments have advantage in extracting features of image with complex boundaries since the V-system contains a great deal of discontinuous basis functions. Therefore, feature extraction of multi-boundary image using V-moments is very promising. This paper performs image retrieval based on their shape features. The results of retrieval experiment, conducted on benchmark database MPEG-7-shape-CE2, show that the algorithm proposed in this paper outperforms some classical moments including Zernike moments, Hu invariant moments, orthogonal Fourier-Mellin moments, Legendre moments and the geometric central moments in retrieval efficiency according to several evaluation indexes.*

Keywords Content based image retrieval; Multi-boundary shape retrieval; Orthogonal moment functions; V-system; V-moment

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Structural index reduction algorithms for differential algebraic equations via fixed-point iteration*

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Abstract

Differential algebraic equations (DAEs) arise naturally in dynamical system modeling, such as electric circuits, mechanical systems and spacecraft dynamics. Generally, these DAEs are large scale nonlinear systems with block structures and high indices which require index reductions for numerical solving. In this paper, motivated by Pryce's structural index reduction method for DAEs, we show the complexity of the fixed-point iteration algorithm and propose a fixed-point iteration method with parameters. It leads to a block fixed-point iteration method which can be applied to large-scale DAEs with block upper triangular structure. Moreover, its complexity analysis is also given in this paper.

Keywords: differential algebraic equations, structural analysis, index reduction, linear programming, fixed-point iteration, block triangular forms.

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A Classcial Hardness Result of R-LWE

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Mingsheng Wang

Abstract

lattice-based cryptography has been developing rapidly, some cryptosystems are based on computational problems of lattices. There are close connections between these computational problems and LWE. Using ring-LWE, we can design more efficient, and more secure cryptosystems. Therefore, it is necessary to further investigate the hardness of the R-LWE problem. Using the conclusions of Gaussian distributions, we give a classical reduction from the most basic computational ideal lattice problem GAPSVP to R-LWE with polynomial modulus.

Computing the Rational Univariate Representations for Zero-dimensional Systems Involving Interval Representation*

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Abstract

The purpose of this paper is to investigate the rational univariate representations of zero-dimensional systems involving the interval representation. Let F be a computable ordered field with real closed extension R. As a representation of algebraic elements over F, the following definition is important for the symbolic computation in real algebra.

Definition 1. Let $u(t) \in F[t]$ be a non-zero univariate polynomial, and a, $b \in F$ with $a < b$. If $u(t)$ has exactly one root Θ in R such that $\Theta \in]a, b[_R$, where $]a, b[_R := \{z \in R \mid a < z < b\}$ is the open interval with endpoints a, b in R, then the only real root Θ of $u(t)$ in $]a, b[_R$ is represented in the form $(u(t), a, b)$. Such a triple $(u(t), a, b)$ is called a real (algebraic) element in its Interval Representation over F.

Definition 1 is a slight generalization of the Interval Representation for real algebraic numbers (i.e. real algebraic elements over the field $\mathbb{Q}$ of rational numbers). For the details of the Interval Representation for real algebraic numbers, refer to §8.5 in [9].

The purpose of this paper is to investigate the rational univariate representations of zero-dimensional systems involving the interval representation. Explicitly speaking, we shall solve the problem as follows:

Problem. Let $(F, \leq)$ be a computable ordered field with real closed extension R, let $\Theta := (u(t), a, b)$ be a real algebraic element in its Interval Representation over F, and let f_1, ..., $f_s \in F[t, x_1, ..., x_n]$ be polynomials over F in $n+1$ variables t, x_1, ..., x_n such that the polynomials $f_1(\Theta, x_1, ..., x_n)$, ..., $f_s(\Theta, x_1, ..., x_n)$ constitute a zero-dimensional system in $F(\Theta)[x_1, ..., x_n]$ (i.e. the system of equations $f_i(\Theta, x_1, ..., x_n) = 0$, $i = 1$, ..., s, has only a finite number of solutions in the algebraic closed field $R(\sqrt{-1})$). Devise an algorithm

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for computing a family of rational univariate representations of the system $\{f_i(\Theta, x_1, ..., x_n) \mid i = 1, ..., s\}$.

A family of rational univariate representations of the system $\{f_i(\Theta, x_1, ..., x_n) \mid i = 1, ..., s\}$ may be formulated in the following definition.

Definition 2. Let the notation be as in the problem above, and χ a new variable. A finite subset $\{[w_i(t, \chi), \phi_{i1}(t, \chi), ..., \phi_{in}(t, \chi)] \mid i = 1, ..., s\}$ of $F[t, \chi] \times F(t, \chi)^n$ is called a family of rational univariate representations of the system $\{f_i(\Theta, x_1, ..., x_n) \mid i = 1, ..., s\}$ (in short, an RUR family of $\{f_i(\Theta, x_1, ..., x_n) \mid i = 1, ..., s\}$), if the following conditions are satisfied for any extension E of F containing R:

(1) For every $i \in \{1, ..., s\}$, $\deg(w_i(\Theta, \chi)) > 0$, $L_i(\Theta) \neq 0$ where $L_i(t)$ is the leading coefficient of $w_i(t, \chi)$ as a polynomial over $F[t]$ in one variable χ, and the denominator of $\phi_{ij}(\Theta, \chi)$ does not vanish at each root of $w_i(\Theta, \chi)$ in E, $j = 1, ..., n$.

(2) If α is a root of $w_k(\Theta, \chi)$ in E for $k \in \{1, ..., s\}$, then $(\phi_{k1}(\Theta, \alpha), ..., \phi_{kn}(\Theta, \alpha))$ is a zero of the system $\{f_i(\Theta, x_1, ..., x_n) \mid i = 1, ..., s\}$ in E^n.

(3) If $(\beta_1, ..., \beta_n)$ is a zero of the system $\{f_i(\Theta, x_1, ..., x_n) \mid i = 1, ..., s\}$ in E^n, then there exist a $k \in \{1, ..., s\}$ and a root α of $u_k(\Theta, \chi)$ in $F(\beta_1, ..., \beta_n)$ such that $(\beta_1, ..., \beta_n) = (\phi_{k1}(\Theta, \alpha), ..., \phi_{kn}(\Theta, \alpha))$.

In this case, we also say that $\{[u_i(t, \chi), \phi_{i1}(t, \chi), ..., \phi_{in}(t, \chi)] \mid i = 1, ..., s\}$ is an RUR family of P modulo Θ (or modulo $(u(t), a, b)$) where $P := \{f_i(t, x_1, ..., x_n) \mid i = 1, ..., s\}$.

To solve the system $\{f_i(\Theta, x_1, ..., x_n) \mid i = 1, ..., s\}$, a natural way is to investigate the extended system $\{u(t), f_1, \cdots, f_s\}$ in $F[t, x_1, ..., x_n]$. However, the two possible cases could happen in practice. Firstly, $\{u(t), f_1, \cdots, f_s\}$ need not be zero-dimensional if $u(t)$ is reducible in $F[t]$. For example, let $\Theta = ((t^2 - 2)(t - 1), \frac{7}{5}, 2)(= \sqrt{2})$, $f_1 = (t - 1)x_1 + (t - 1)x_2$, and $f2 = (t - 1)x_1 - (t - 1)x_2$, Obviously, the system $\{f_1(\Theta, x_1, x_2), f_2(\Theta, x_1, x_2)\}$ is zero-dimensional, but $\{(t^2 - 1)(t - 1), f_1(t, x_1, x_2), f_2(t, x_1, x_2)\}$ is not zero-dimensional. Secondly, even if the rational univariate representation of the extended system is obtained, not only this representation is more complicated, but the specified value $t = \Theta$ must be considered in a further discussion.

In this paper, an effective method is presented for deciding whether or not a system in $F(\Theta)[x_1, ..., x_n]$ is zero-dimensional. As a main result in this paper, we give an algorithm for computing the rational univariate representations of zero-dimensional systems in $F(\Theta)[x_1, ..., x_n]$. The technique in this paper is to compute triangular decompositions of polynomial systems. Hence, our algorithm does not involve the Gröbner bases calculation. With the aid of the computer algebraic system **Maple**, a general program has been made to compute rational univariate representations of zero-dimensional systems in $F(\Theta)[x_1, ..., x_n]$ when $F = \mathbb{Q}$, the field of rational numbers. It should be pointed out that the algorithm in this paper does not preserve the multiplicities of solutions as in [6,13]. It is a repayment that our rational univariate representations are simpler.

In the final section of this paper, several examples are given to illustrate the efficiency of our algorithm.

Vincent 定理的多元推广*

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摘要 Vincent 定理指出: 给定 d 次实系数多项式 $f(x)$ 和开区间 $(a_1,\ b_1)$. 多项式 $f(x)$ 在区间 $(a_1,\ b_1)$ 上没有实根, 当且仅当存在正常数 δ, 对任意区间 $(a,\ b)\subset(a_1,\ b_1)$ 满足 $|a-b|<\delta$ 时, 使得多项式 $(1+x)^d f(\frac{a+bx}{1+x})$ 的系数不变号 (都是正数或都是负数). 本文的主要工作是推广这一结果到一般的多变元代数系统. 实系数多项式 $f\in\mathbb{R}[x_1,\ldots,x_n]$. f 相对于变元 x_i 的次数记为 d_i. 区间的笛卡尔积 $I=[a_1,b_1]\times\cdots\times[a_n,b_n]$ (也称为 Box), 记 $\phi(I)=\max\{b_i-a_i,\ i=1,\ldots,n\}$. 定义

$$f_I=(1+x_1)^{d_1}\cdots(1+x_n)^{d_n}f(\frac{a_1+b_1x_1}{1+x_1},\ldots,\frac{a_n+b_nx_n}{1+x_n}),$$

f_I 称为 f 相对于 Box I 的伴随多项式. 本文证明了: 对给定的多项式 $f_1,\ldots,f_m\in\mathbb{R}[x_1,\ldots,x_n]$, 且 Box $\Lambda\subset\mathbb{R}^n$. 方程组 $\{f_1=0,\ldots,f_m=0\}$ 在 Box Λ 上没有零点, 当且仅当存在正常数 δ (与 Box Λ 有关), 对任意 Box $I\subset\Lambda$, 当 $\phi(I)<\delta$ 时, 使得伴随多项式

$$f_{1I},\ldots,f_{mI}$$

中至少有一个 f_{iI} 的系数全是正 (或负) 数且 f_i 在 Box I 的所有顶点上的值不为 0.

关键词 Vincent 定理, 代数系统, 实零点.

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Linear Invariant Generation for Safety Verification of Nonlinear Systems Using Conservative Approximation

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As one of most important research issues in formal analysis of dynamical systems, safety verification aims to decide if there exists a trajectory starting from the initial set that reaches some unsafe region in the state space. In this paper, we will apply the invariant generation method to verify safety of nonlinear systems. Rather than studying the complex nonlinear systems directly, we first present a linear approximation method to transform the general nonlinear systems into the associated linear ones, then suggest a symbolic method to obtain linear invariants, which guarantee the safety property of the resulting linear systems. Some experiments are provided to illustrate the efficiency of our method.

ON THE LINEAR INDEPENDENCE AND PARTITION OF UNITY OF ARBITRARY DEGREE ANALYSIS-SUITABLE T-SPLINES*

JINGJING ZHANG · XIN LI

Abstract Analysis-suitable T-splines are a topological restricted subset of T-splines, which are optimized to meet the needs for design and analysis. The paper independently derives a class of bi-degree (d_1, d_2) T-splines for which no perpendicular T-junction extensions intersect, and provides a new proof for the linearly independence of the blending functions. We also prove that the sum of the basis functions is one for an AS T-spline if the T-mesh is admissible based on a recursive relation.

Keywords T-splines, analysis-suitable T-splines, linear independence, partition of unity, isogeometric analysis.

Conformal Parameterization Based Tool-path Planning for Meshes

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The similar property of conformal parameterization makes it able to locally preserve the shapes between surface and its parameter domain, as opposed to common parameterization methods. A parametric tool-path planning method is proposed in this paper through such parameterization of triangular meshes which is furthermore based on the geodesic on meshes. The parameterization has the properties of local similarity and free boundary which are exploited to simplify the formulas for computing path's parameters, which play a fundamentally important role in tool-path planning, and keep the path boundary-conformed and smooth. Experimental results are given to illustrate the effectiveness of the proposed methods, as well as the error analysis.

误差可控计算软件 Isreal

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算术表达式的精确计算是确保各类复杂计算正确性的基础. 然而，由于计算过程中的舍入误差，现有的计算软件或计算设备经常会出现不可接受，甚至完全错误的计算结果. 比如，对于$20^{65}-e^{65\times\ln(20)}$，正确结果是 0，可是 Matlab (R2012a) 的计算结果却是6.1253×10^{70}. 引起这类错误的原因是不正确的舍入：现有的计算方式是采用统一的、不变的"有效位数"模式，即计算前要设定有效位数，然后所有中间运算均按照这个有效位数进行舍入，当然结果也保留相同的有效位数. 但是，这样计算得来的最终结果很可能没有 1 位有效数字正确.

基于这个原因，许多软件在计算之前留给用户"太多的"参数设置. 例如 Maple 的 Digits、Matlab 的 digits、Mathematica 的$MaxExtraPrecision、Pari 的\p、Gmp 的 mpf_set_default_prec 等等，包括区间运算软件也不例外，比如 iRRAM 的 setRwidth、Range software of Oliver Alberth 的 n 等. 若上述这些参数设置错误，则不会得到正确结果，但是在计算之前用户并不知道如何设置它们. 因此，正如文献所说："不幸的是，绝大多数数值计算软件工具不能给用户提供任何精度保证(Unfortunately, most numerical software tools do not provide any accuracy guarantee to the user)"或"只能保证一个原子运算的正确舍入(it only guarantees correct rounding for an atomic operation)".

鉴于目前状况，本文作者设计开发了一个误差可控计算软件 Isreal(下载网址：http://www.zhaoshizhong.org/download.htm). 任给一个算术表达式，只要用户输入需要保留的小数位数，Isreal 就给出相应的正确结果. 不论计算模型是"良态的"还是"病态的"，计算结果始终与理论结果相吻合，即始终是正确的.

NURBS CURVE INTERPOLATION ALGORITHM BASED ON S-CURVE ACC/DEC CONTROL METHOD

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This paper proposes a look-ahead interpolation scheme for short line segments. The proposed interpolation method consists of two modules: NURBS fitting and S-curve acceleration/deceleration (ACC/DEC) feedrate-planning modules. Depending on the length and the CSB criterion, these linearized segments can be regarded as noise, continuous short blocks (CSBs), or G01 blocks. The junctions are located where the curvature beyond the NURBS setting value or undergoing violent changes and the ending points of the long straight segments(non-CSB). The NURBS fitting module first looks ahead several CSBs and converts them into parametric curves in real-time machining. It can ensure that the position, slope, and curvature at the junctions of the parametric curves, and unfitted line segments are all continuous; Then the acc/dec feedrate-planning module proposes a new algorithm to determine the feedrate at the junction of the fitting curve and unfitted short segments, and the corner feedrate within the fitting curve. Simulations and experiments show that the implemented NURBS cutting can significantly improve machining accuracy and reduce cutting time to satisfy the requirements of today's high-speed and –accuracy machining.

Weighted-average alternating minimization algorithm and its application to magnetic resonance image reconstruction based on compressive sensing

Yonggui Zhu, Hao Li and Xiang Bi

Abstract

The problem of compressive-sensing (CS) L2-L1-TV reconstruction of magnetic resonance (MR) scans from undersampled k-space data has been addressed in numerous studies. However, the regularization parameters in models of CS L2-L1-TV reconstruction are rarely studied. Once the regularization parameters are given, the solution for an MR reconstruction model is fixed and is less effective in the case of strong noise. To overcome this shortcoming, we present a new alternating formulation to replace the standard L2-L1-TV reconstruction model. We prove that this new formulation is equivalent to the standard one in some conditions. A weighted-average alternating minimization method is proposed based on this new formulation and a convergence analysis of the method is carried out. The advantages of and the motivation for the proposed alternating formulation are explained. Experimental results demonstrate that the proposed formulation yields better reconstruction results in the case of strong noise and can improve image reconstruction via flexible parameter selection.

Keywords: Compressive sensing; alternating minimization method; weighted average; magnetic resonance image reconstruction.

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