



# 第六期符号计算暑期讲习班

SSSC 2019

# 会议手册

中国·重庆

2019年7月21日-27日

## 大会主席

王东明 北京航空航天大学/广西民族大学

## 程序委员会

陈发来 中国科学技术大学

冯 勇 中国科学重庆绿色智能技术研究院

冯如勇 中国科学院数学与系统科学研究院（共同主席）

侯庆虎 天津大学

余志坤 北京航空航天大学

王东明 北京航空航天大学/广西民族大学

夏壁灿 北京大学

支丽红 中国科学院数学与系统科学研究院（共同主席）

## 组织委员会

陈长波 中国科学重庆绿色智能技术研究院

吴文渊 中国科学重庆绿色智能技术研究院

杨 静 广西民族大学

杨文强 中国科学重庆绿色智能技术研究院

## Arrangement

会议地点：三楼鸿图鸿业厅

用餐地点：三楼鸿福鸿禧厅

备注：21 号注册 14:30-18:30

21 日-26 日晚餐：18:00-20:30

	July 22	July 23	July 24	July 25
8:50-9:00	Opening Remarks			
9:00-11:00	Li-Yong Shen	Li-Yong Shen	Shaoshi Chen	Zhengfeng Yang Ming Xu
11:00-11:15	Tea Break			
11:15-12:15	Li-Yong Shen	Li-Yong Shen	Frederic Chyzak	Tsung-Lin Lee
12:15-14:00	Lunch			
14:00-16:00	Shaoshi Chen	Shaoshi Chen	Mingsheng Wang	Mingsheng Wang
16:00-16:30	Tea Break			
16:30-18:30	Mohab Safey EI Din	Mohab Safey EI Din	Jin-San Cheng (one hour)	Xiaoxian Tang (one hour)
18:30-20:00	Dinner			

	July 26	July 27
9:00-11:00	Mingsheng Wang	Free Discussion
11:00-11:15	Tea Break	
11:15-12:15	Chenqi Mou	
12:15-14:00	Lunch	
14:00-15:00	Dingkang Wang	
15:00-15:15	Tea Break	
15:15-16:15	Jinwang Liu	
16:15-16:30	Closing Remarks	
18:30-20:00	Dinner	

# Title and Abstract

---

## Introduction to Symbolic Integration and Summation

Shaoshi Chen  
AMSS, Chinese Academy of Sciences

Symbolic integration and summation is a classical and fundamental topic in Symbolic Computation with applications in Combinatorics and Mathematical Physics. In this tutorial, we will overview some basic algorithms of symbolic integration and summation, such as Risch's algorithm, Gosper's algorithm and Zeilberger's algorithm. In particular, we will explain why elementary functions  $\exp^x$  and  $1/\log(x)$  have no elementary indefinite integrals and how to prove combinatorial identities algorithmically.

## Certified Numerical Real Root Isolation for Bivariate Polynomial Systems

Jin-San Cheng  
AMSS, Chinese Academy of Sciences

In this paper, we present a new method for isolating real roots of a bivariate polynomial system. Our method is a subdivision method which is based on real root isolation of univariate polynomials and analyzing the local geometrical properties of the given system. We propose the concept of the orthogonal monotone system in a box and use it to determine the uniqueness and the existence of a simple real zero of the system in the box. We implement our method to isolate the real zeros of a given bivariate polynomial system. The experiments show the effectivity and efficiency of our method, especially for systems with high degrees and sparse terms. Our method also works for non-polynomial systems. It is a joint work with Junyi Wen.

## Generalized Hermite Reduction, Creative Telescoping and Definite Integration of D-Finite Functions

Frédéric Chyzak  
INRIA, France

Hermite reduction is a classical algorithmic tool in symbolic integration. It is used to decompose a given rational function as a sum of a function with simple poles and the derivative of another rational function. In this talk, we extend Hermite reduction to arbitrary linear differential operators instead of the pure derivative, and develop efficient algorithms for this reduction. We then apply our generalized Hermite reduction to the computation of linear operators satisfied by single definite integrals of D-finite functions of several continuous or discrete parameters. The resulting algorithm is a generalization of reduction-based methods for creative telescoping.

(Joint work with A. Bostan, P. Lairez, and B. Salvy.)

## Computing the Numerical Rank of Large Matrices

Tsung-Lin Lee  
National Sun Yat-sen University

If rank deficiency for a large matrix is known to be small apriori, there seems no need to compute SVD to find all singular values for determining its rank. While the Householder QR with column pivoting algorithm proposed by Golub and Businger in 1965 works quite well in general for this purpose, there exist counter examples (by W. Kahan) that the method would fail. These failures of this sort had been overcome by T. Chan's RRQR algorithm in 1989. In practice, such as in signal processing, rank updatings and downdatings occur commonly along with the

rank-revealing. While the UTV decomposition method for rank revealing proposed by G.W. Stewart in 1992 works quite well in rank updatings, it may not be efficient for the downdatings. Recently, an efficient algorithm for rank revealing is proposed. For the method, both updatings and downdatings become quite straightforward. The related results will also be presented in this talk.

### **Factorization and Equivalence on Multivariate Polynomial Matrices**

Jinwang Liu

Hunan University of Science and Technology

A multidimensional  $(n - D)$  system can be represented by a  $n - D$  polynomial matrix, we firstly investigate the general prime factorizations of  $n - D$  polynomial matrices: zero prime factorization, minor prime factorization, factor prime factorization. Second, we investigate the equivalence of  $n - D$  polynomial matrix. For a class of  $n - D$  polynomial matrices, giving some tractable necessary and sufficient conditions under which matrices are equivalent to their Smith-form, there conditions are easily checked by computing the reduced Gröbner bases of some ideals. Finally, we investigate  $n - D$  polynomial matrix equivalence recursive problem, i.e.,  $\text{diag}(1, B) \sim \text{diag}(1, 1, C)$ ,  $B$  is or not equivalent to  $\text{diag}(1, C1)$ , a negative answer is obtained.

### **On the Connection Between Lexicographic Groebner Bases and Triangular Sets**

Chenqi Mou

Beihang University

Lexicographic Groebner bases and triangular sets are standard tools in polynomial elimination theory. In this talk, we present new results on the intrinsic structures of lexicographic Groebner bases and relations between lexicographic Groebner bases and the minimal triangular sets contained in them called W-characteristic sets. It is shown that either this W-characteristic set is normal or there are explicit (pseudo-)divisibility relations between the polynomials in it. Based on these properties of W-characteristic sets, we introduce the concept of characteristic pair consisting of a reduced lexicographic Groebner basis and its normal W-characteristic set, and design an algorithm for decomposing any polynomial set into finitely many characteristic pairs with associated zero relations, which provide representations of the zero of the polynomial set in terms of those of Groebner bases and in terms of those of triangular sets simultaneously. Several nice properties of the decomposition and the resulting characteristic pairs, in particular relationships between the Groebner basis and the W-characteristic set in each pair, are established.

This talk is based on joint work with Dongming Wang and Rina Dong.

### **Some efficient algorithms in real algebraic geometry**

Mohab Safey El Din

Laboratoire d'Informatique de Paris 6, France

Real solution sets to polynomial systems of equations and inequalities with real coefficients and maps between these sets are the core objects of real algebraic geometry. Algorithms in real algebraic geometry are designed to study real solutions to polynomial systems and find applications in many areas of engineering sciences.

This course will overview the most recent developments of computer algebra algorithms for real solving polynomial systems. It will be subdivided in three parts. First, we will introduce the notions of real and real closed fields and algorithms for univariate solving. Next, we will switch to multivariate situations in order to compute sample points in real solution sets to polynomial systems. Finally, we will study algorithms for performing quantifier elimination over the reals.

## 有理曲线与曲面的syzygy 理论及应用进展

申立勇  
中国科学院大学

有理曲线与曲面的 syzygy 类似于曲线曲面表示射影空间的补空间，它的性质往往能够反馈曲线曲面本身的性质，但是 syzygy 具有次数低，便于计算等优点，因此曲线与曲面的 syzygy 研究是有意义的内容。特别地， $\mu$  基是一种特殊形式的 syzygy 表示，能够联结其参数表示与隐式表示之间的桥梁，是一种新的曲线与曲面的表达形式。它承载了曲线与曲面的所有内蕴几何性质，曲线  $\mu$  基次数低、计算快，为隐式化、点逆公式表达及奇异点计算等应用提供高效算法。曲面  $\mu$  基在部分曲面的有类似结果，但是通用曲面  $\mu$  基的研究依然有待扩展，有时候可以通过 syzygy 本身求的隐式方程更为便捷。本课程将介绍近年来 syzygy 和  $\mu$  基理论在平面有理曲线、空间有理曲面、特殊有理曲面、一般有理曲面上的主要研究进展、 $\mu$  基理论在计算机辅助几何设计中包括快速隐式化、点逆公式表达及奇异点计算在内的几个重要应用。

## Real Root Classification and Its Applications

Xiaoxian Tang  
Texas A&M University, USA

Classifying parameters according to numbers of real solutions of a general zero-dimensional polynomial system is a fundamental problem in computational real algebraic geometry. In this talk, we give an introduction to a standard method of real root classification and many applications in system biology and statistics.

## Comprehensive Groebner System: Algorithms and Applications

Dingkang Wang  
AMSS, Chinese Academy of Sciences

We will introduce the definition of comprehensive Groebner System (CGS) and present some efficient algorithms to compute the CGS of a parametric polynomial system. We will also give some applications using CGS, including solving system of parametric polynomial equations, discovering geometric theorems automatically, performing quantifier elimination over an algebraic closed field and computing greatest common divisors of multivariate polynomials with parameters.

## Groebner 基快速计算的理论与算法

王明生  
中国科学院信息工程研究所

本课程主要讲解计算Groebner基的GVW算法的理论基础，及未来发展的一些想法。

## Efficient Algorithms for Solving Poly-Power Constraints

Ming Xu  
East China Normal University

We first consider a class of univariate real functions—poly-powers—that extend integer exponents to real algebraic exponents for polynomials. Our purpose is to isolate positive roots of such a function into disjoint intervals, each contains exactly one positive root and together contain all, which can be easily refined to any desired precision. To this end, we classify poly-powers into simple and non-simple ones, depending on the number of linearly independent exponents. For the former, based on Gelfond–Schneider theorem, we present two complete isolation algorithms—exclusion and differentiation. For the latter, their completeness depends on Schanuel’s conjecture.

We then studies the satisfiability problem of poly-power constraints that are conjunctions of poly-power equations and inequalities. To solve the poly-power constraint, we present a sound and complete procedure that incorporates conflict-driven learning with the exclusion algorithm for isolating positive roots of poly-powers. Furthermore, we introduce a kind of optimal interval-splitting, based on the Stern–Brocot tree and on binary rational numbers respectively, so that the operands occurring in the execution are chosen to be as simple as possible. The solving procedure, thereby, turns out to be promisingly efficient on randomly generated examples.

## **Safety Verification of Nonlinear Hybrid Systems Based on Bilinear Programming**

Zhengfeng Yang  
East China Normal University

In safety verification of hybrid systems, barrier certificates are generated by solving the verification conditions derived from non-negative representations of different types. In this talk, we present a new computational method, sequential linear programming projection, for directly solving the set of verification conditions represented by the Krivine-Vasilescu-Handelman’s positivstellensatz. The key idea is to decompose it into two successive optimization problems that refine the desired barrier certificate and those undetermined multipliers, respectively, and solve it in an iterative scheme. The most important benefit of the proposed approach lies in that it is much more effective than the LP relaxation method in producing real barrier certificates, and possesses a much lower computational complexity than the popular sum of square relaxation methods, which is demonstrated by the theoretical analysis on complexity and the experiment on a set of examples gathered from the literature.

## 酒店位置

地址：重庆市渝中区中山三路 139 号





交通路线

<div>希尔顿酒店——江北机场 T2 航站楼</div> <div><div>票价¥6 无堵车风险 轨道交通3号线</div><div>1小时6分钟   29.6公里   步行660米</div></div> <div><div><div>● 希尔顿酒店(重庆店)</div><div>步行 400米</div></div><div><div>两路口站 上车</div><div>轨道交通3号线 (江北机场t2航站楼方向)</div><div>江北机场T2航站楼站 (4口出) 下车</div><div>步行 250米</div></div><div><div>● 重庆江北国际机场-T2航站楼</div></div></div>	<div>希尔顿酒店——江北机场 T3 航站楼</div> <div><div>票价¥6 无堵车风险 轨道交通3号线 → 轨道交通10号线</div><div>1小时1分钟   27.3公里   步行850米</div></div> <div><div><div>● 希尔顿酒店(重庆店)</div><div>步行 400米</div></div><div><div>两路口站 上车</div><div>轨道交通3号线 (江北机场t2航站楼方向)</div><div>重庆北站南广场站 下车</div><div>站内换乘 步行 320米</div><div>重庆北站南广场站 上车</div><div>轨道交通10号线 (王家庄方向)</div><div>江北机场T3航站楼站 (2口出) 下车</div><div>步行 140米</div></div><div><div>● 重庆江北国际机场T3航站楼</div></div></div>
<div>希尔顿酒店——重庆北站北广场</div> <div><div>票价¥3 最佳 轨道交通3号线 → 轨道交通10号线</div><div>40分钟   10.9公里   步行720米</div></div> <div><div><div>● 希尔顿酒店(重庆店)</div><div>步行 400米</div></div><div><div>两路口站 上车</div><div>轨道交通3号线 (江北机场t2航站楼方向)</div><div>重庆北站南广场站 下车</div><div>站内换乘 步行 320米</div><div>重庆北站南广场站 上车</div><div>轨道交通10号线 (王家庄方向)</div><div>重庆北站北广场站 下车</div><div>步行 10米</div></div><div><div>● 重庆北站北广场</div></div></div>	<div>希尔顿酒店——重庆西站</div> <div><div>票价¥5 最佳 轨道交通1号线 → 473路</div><div>1小时9分钟   17.9公里   步行1.3公里</div></div> <div><div><div>● 希尔顿酒店(重庆店)</div><div>步行 440米</div></div><div><div>两路口站 上车</div><div>轨道交通1号线 (尖顶坡方向)</div><div>高庙村站 (1口出) 下车</div><div>步行 570米</div><div>联芳花园站 上车</div><div>473路</div><div>重庆西站 下车</div><div>步行 310米</div></div><div><div>● 重庆西站</div></div></div>

# 重庆周边

## 1. 网红路线

希尔顿酒店——两路口轻轨站（皇冠大扶梯）——李子坝轻轨站（李子坝观景平台）——解放碑步行街——八一路好吃街——长江索道——洪崖洞——朝天门码头（两江游船）

希尔顿酒店——磁器口古镇——歌乐山

希尔顿酒店——中山四路——桂园——周公馆——三峡博物馆——人民大礼堂

## 2. 夜景路线

希尔顿酒店——壹华里夜景公园——枇杷园食为鲜火锅(金竹村店)

希尔顿酒店——南山一棵树观景台——南山老幺泉水鸡