Computing the greatest common divisor of several parametric univariate polynomials via generalized subresultants

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Abstract

In this talk, we tackle the following problem: compute the gcd for *several* univariate polynomials with *parametric* coefficients. It amounts to partitioning the parameter space into "cells" so that the gcd has a uniform expression over each cell and constructing a uniform expression of gcd in each cell. We tackle the problem as follows. We begin by making a natural and obvious extension of subresultants of two polynomials to several polynomials. Then we develop the following structural theories about them.

1. We generalize Sylvester's theory [1] to several polynomials, in order to obtain an elegant relationship between generalized subresultants and the gcd of several polynomials, yielding an elegant algorithm.
2. We generalize Habicht's theory [2] to several polynomials, in order to obtain a systematic relationship between generalized subresultants and pseudo-remainders, yielding an efficient algorithm.

Using the generalized theories, we present a simple (structurally elegant) algorithm which is significantly more efficient (both in the output size and computing time) than algorithms based on previous approaches.

References

[1] J. Sylvester, On a theory of the syzygetic relations of two rational integral functions, comprising an application to the theory of Sturm's functions, and that of the greatest algebraical common measure. *Philosophical Transactions of the Royal Society of London* **volume**(143), 407–548 (1853).

[2] W. Habicht, Eine Verallgemeinerung des Sturmschen Wurzelzählverfahrens. *Commentarii Mathematici Helvetici* **volume**(21), 99–116 (1948).