Non Linear q-difference Equations and Curvatures

(joint work with L. Di Vizio)

Charlotte Hardouin
Institut de mathématiques de Toulouse, France

We show that the results in the talk of L. Di Vizio combined with [DV02] and [Hen96], allow to describe the generic (algebraic and differential) Galois groups of a complex q-difference equation, with $q \in \mathbb{C} \setminus \{0, 1\}$, by means of curvatures (but not the same curvatures according that q is a root of unity; an algebraic number, not a root of unity; or a transcendental number).

There are many Galois theories for q-difference equations defined over fields such as $\mathbb{C}$, the field of elliptic functions, or the differential closure of $\mathbb{C}$. We give the comparison results between the two generic Galois groups that we have introduced and the other Galois groups in the literature, especially with the Hardouin-Singer Galois group. In particular, we show that the differential algebraic relations between the solutions of the equation are endowed by the differential algebraic relations satisfied by the curvatures.

Inspired by the work of B. Malgrange for non linear differential equations, A. Granier attach to a non linear q-difference equation $Y(qx) = F(Y(x))$ with $Y \in Gl_\nu$, a D-groupoid, *i.e.* , a differential ideal in the space of jets of the analytic variety $P^1\mathbb{C} \times \mathbb{C}^\nu$. The Malgrange-Granier D-groupoid was recently used by G. Casale and J. Roques to prove some non-integrability results and for a linear q-difference system with constant coefficients, A. Granier was able to show that the D-groupoid coincides with the usual Galois group of the system.

The arithmetic description of the differential generic Galois group plus the results of A. Granier imply that, in the linear case, the Malgrange-Granier D-groupoid of a linear q-difference system essentially coincides with the Kolchin closure of the dynamic of the system and that the group that fix the transversals in the Malgrange-Granier D-groupoid is the generic differential Galois group introduced in the previous talk. This result is the first attempt to relate the D-groupoid of B. Malgrange and the linear differential algebraic groups of Kolchin.

References

[DV02] Lucia Di Vizio. Arithmetic theory of q-difference equations. The q-analogue of Grothendieck-Katz's conjecture on p-curvatures. *Inventiones Mathematicae*, 150(3):517–578, 2002. arXiv:math.NT/0104178.

[Hen96] Peter Hendriks. *Algebraic Aspects of Linear Differential and Difference Equations.* PhD thesis, University of Groningen., 1996.