Parametrizing Linear Systems

Daniel Robertz
RWTH Aachen University

Given a system of (homogeneous) linear equations, an adequate way to represent the space of solutions is as the image of an operator to be constructed from the equations. In case the equations have coefficients in a field, Gaussian elimination achieves this objective. If the system is underdetermined, then the corresponding Gauss-reduced matrix singles out some variables of the system as parameters and specifies how all other variables are expressed (linearly) in terms of the parameters. This procedure can be viewed as identifying the kernel of the linear map induced by the system matrix as the image of another linear map.

More generally, if the equations have coefficients in a ring, that is not necessarily a (skew-) field, the question arises whether it is still possible to construct an operator which is defined over the same ring and whose image equals the space of solutions. For instance, a system of (homogeneous) linear partial differential equations may be written as an equation whose left hand side is a matrix differential operator applied to the vector of unknown functions and whose right hand side is zero. Is it possible to *parametrize* the system, i.e. to construct another matrix differential operator whose image equals the kernel of the given one? In general, the answer is negative.

This talk presents recent joint work with F. Chyzak and A. Quadrat addressing several aspects of the above problem, in particular the algorithmic decision of existence and the construction of parametrizations for the case of systems of linear partial differential equations (with constant, polynomial, rational, or analytic coefficients).

References

[1] F. Chyzak, A. Quadrat, and D. Robertz. Effective algorithms for parametrizing linear control systems over Ore algebras. *Applicable Al-*

gebra in Engineering, Communication and Computing, 16(5):319–376, 2005.

[2] J.-F. Pommaret. *Partial differential control theory*. Kluwer, 2001.

[3] A. Quadrat and D. Robertz. Computation of bases of free modules over the Weyl algebras. *Journal of Symbolic Computation*, 42(11–12):1113–1141, 2007.

[4] A. Quadrat and D. Robertz. Baer's extension problem for multidimensional linear systems. In *Proceedings of the 18th International Symposium on Mathematical Theory of Networks and Systems (MTNS 2008), Virginia Tech, Blacksburg, Virginia (USA)*, 2008.

[5] A. Quadrat and D. Robertz. Controllability and differential flatness of linear analytic ordinary differential systems. In *Proceedings of the 19th International Symposium on Mathematical Theory of Networks and Systems (MTNS 2010), Budapest, Hungary*, 2010.