About the Lie Algebras of Differential Operators on A Path Algebra

Fang Li
Zhejiang University, China

The aim of this article is to study the Lie algebra $\mathfrak{Diff}(\mathbf{k}\Gamma)$ of differential operators on the path algebra $\mathbf{k}\Gamma$ of a quiver Γ and relate this Lie algebra to the algebraic and combinatorial properties of $\mathbf{k}\Gamma$. We first characterize when a linear operator on a path algebra is a differential operator and thus obtain a standard basis of $\mathfrak{Diff}(\mathbf{k}\Gamma)$.

Moreover, mainly, we show that the Lie algebra $\mathfrak{OutDiff}(\mathbf{k}\Gamma)$ of outer differential operators, defined to be the quotient of $\mathfrak{Diff}(\mathbf{k}\Gamma)$ modulo $\mathfrak{InDiff}(\mathbf{k}\Gamma)$, is closely related to the topological and graph theoretic properties of a finite connected planar Γ, such as the genus 0 and Euler's characteristic of the Riemann sphere which the quiver Γ is embedded into rather than into the plane. In particular, from a careful analysis of the conection matrix and boundary matrix of a quiver, a canonical basis of $\mathfrak{OutDiff}(\mathbf{k}\Gamma)$ is given.