# Differentially Flat Nonlinear Control Systems: An Overview of The Theory and Applications, and Differential Algebraic Aspects

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Motion planning and trajectory tracking are among the main problems in Control Theory. These problems, though non trivial even in the class of linear systems, receive a simple solution in the particular class of systems called "Differentially flat".

For a finite dimensional system described by a set of differential equations, controlled by m independent inputs, it is called differentially flat if, and only if, there exists a vector of differentially independent generalized output (i.e. whose components are functions of the state, input variables and a finite number of their time derivatives) of the same dimension as the input vector, such that every system variable may be expressed as a function of this output and a finite number of its derivatives. Such an output, if it exists, is called "flat output". This definition may be rigorously expressed in the framework of manifolds of jets of infinite order and Lie-Bäcklund isomorphisms. As a direct consequence of this definition, all the trajectories generated by such a system may be parametrized by flat output ordinary curves in a suitable vector space (thus geometrically flat), without need to integrate the differential equations of the system. Moreover, it can be shown that the flatness property is equivalent to "endogeneous dynamic feedback linearizability", a property which means that the system may be transformed by Lie-Bäcklund isomorphism into a linear controllable system. The latter property is most helpful when one wants to stably track a reference trajectory by feedback, using standard linear control design methods.

The practical interest and simplicity of this approach, the so-called "flatness-based design", will be demonstrated by videos of various oscillating mechanical systems, showing how their oscillations may be significantly and robustly attenuated. Note also that, though non generic, differentially flat systems are often encountered in industrial applications

A major difficulty related to such systems concerns the determination of flat outputs if they exist. Computable necessary and sufficient conditions for differential flatness have been obtained, in the framework of module theory on non commutative polynomial rings of the operator $d/dt$. A system is differentially flat if and only if its variational system module admits an integrable basis, more precisely a basis of polynomial one-forms that admits a unimodular integrating factor. This integrating factor is determined by solving the so-called "generalized moving frame structure equations".

Computer algebra sequential procedures will be presented to compute its solutions, directly giving the flat outputs of the system if they exist. Simple examples, showing the computational complexity, and open problems will be presented, mostly related to the finiteness of these procedures.

## References


[1] J. Lévine, Analysis and Control of Nonlinear Systems: A Flatness-based Approach, *Mathematical Engineering Series*, Springer, 2009.

[2] J. Lévine, On Necessary and Sufficient Conditions for Differential Flatness, to appear in *Applicable Algebra in Engineering, Communication and Computing.*