# Matrix representations for generalized term orders

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Dealing with polynomial rings in computer algebra is strongly connected with the concept of Gröbner bases due to the fact that they are understood to provide a possibility of performing algorithms suitable for most essential computations in polynomial rings. However a big drawback is the time complexity which in the worst case can be double exponential in the number of solutions of the system of equations in concern. In fact, the complexity depends on the term order used for the computations. In applications it is sometimes favorable to use a so-called elimination order to compute a Gröbner bases which makes it easy to solve the given system of equations. Unfortunately especially these orders usually lead to more complex computations than graded orders. By introducing concepts form combinatorics and polyhedral geometry the theory was advanced by creating the concepts of Gröbner fan and Gröbner walk which are used for converting a given Gröbner basis to one with respect to a different term order. The point is that for certain cases it can be more efficient to compute a Gröbner basis with respect to a graded term order and transform it to a Gröbner basis with respect to an elimination order using the Gröbner walk than to compute the later directly using Buchberger's algorithm. The trigger for these developments was Robbiano's classification of term orders stating that every term order can be represented by a matrix over the reals [Rob85]. In rings of difference-differential operators there are several approaches to compute Gröbner bases. Zhou and Winkler [ZW06, ZW08] extended techniques used for Gröbner basis computations in Laurent polynomial rings including the concept of generalized term orders in order to provide a generalized view of the situation. A crucial point for reasoning about the comlexity of their algorithm is a better understanding of generalized term orders. We are going to show that for any generalized term order on the set of difference-differential terms $[\Delta, \Sigma] = [\delta_1, \ldots, \delta_m, \sigma_1, \ldots, \sigma_n]$ there exists $t > m + n$, a term order on the set of terms $[X] = [x_1, \ldots, x_t]$ and a map $\phi : [\Delta, \Sigma] \to [X]$ such that the order of any two difference-differential terms is preserved by $\phi$. Hence, given $\phi$ and the matrix representing the term order on $[X]$ we have a representation of the generalized term order on $[\Delta, \Sigma]$.

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