Differential Algebraic Groups and Factorization of Partial Differential Operators

(joint work with Phyllis Cassidy)

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An ordinary differential operator can be factored as a product of irreducible operators and any two such factorizations have the same number of factors and, after a possible permutation, these factors are equivalent in a suitable sense. Examples showing that such a result is not true for partial differential operators have been known for over 100 years.

Solutions of systems of homogeneous linear partial differential equations form a group under addition and are an example of a differential algebraic group. We show that a Jordan-Hölder type theorem holds for such groups, that is, any such group can be filtered by a finite subnormal series of differential algebraic groups such that successive quotients are "almost simple". Furthermore, any two such series have the same length and, after a possible permutation, successive quotients are "isogenous". This allows us to recover a version of unique factorization for partial differential operators. Many examples will be shown.