Rational Curves and Differential Equations *

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Abstract

The solutions of a non-linear ordinary differential equation (ODE) have been studied from geometric point of view. Recently, it have been systematically developed by R. Feng and X-S. Gao in their papers on rational general solutions of an autonomous ODE of order 1 ([FG04], [FG06]). We have continued this approach to study the rational general solutions of a parametrizable non-autonomous ODE of order 1 ([NgoW]). This is a natural extension of the autonomous ODEs of order 1 with rational solutions and it leads to studying a system of autonomous ODEs of order 1 and of degree 1. We call the associated system of the original ODE. It turns out that an autonomous ODE with rational solutions has a simple associated system of ODEs. In fact, it is again an autonomous ODE in one indeterminate of order 1 and of degree 1 and we already had a degree bound for its rational solutions. However, we do not have a degree bound for rational solutions of the associated system in general.

The associated system is a planar rational system, whose rational solutions form rational parametrization of the rational invariant algebraic curves of the corresponding polynomial system. The problem is how to find an effective degree bound for its rational solutions. This problem is known as Poincaré problem and it has been solved in the case without dicritical singularities.

The presentation is to describe the whole process and some open questions on finding rational solutions of non-linear algebraic ODEs of order 1, especially on finding rational solutions of the associated system.

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