BLENDING QUADRIC SURFACES VIA BASE CURVE

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Abstract

A method for blending surfaces (implicit or parametric) is introduced. The blending surface is defined by a collection of curves generated through the same base curve and has a parametric representation. Here the given surfaces are not restricted to any particular type of surface representation as long as they have a well-defined and continuous normal vector at each point of their boundaries, where are to be blended. In this paper, we mainly discussed the blending problems of quadratic surfaces. In particular, we derive the uniform parametric blending surface of some quadratic surfaces(6 close quadratic surfaces) at the first time. This method also can solve n-way quadratic close surfaces joints. To some special kind of surfaces, we can get higher order continuity blending surfaces. The method is extensible to blend general surfaces, although we concentrate on quadratic surfaces.

Key words: surface blending, base curve, parametric surface, normal plane

1 Introduction

One of the fundamental tasks of CAGD is surface blending. There are several methods to solve the problem. For example, Hoffmann and Hopcroft proposed the potential method in 1986 (see [7]); Warren proposed the ideal theory method in 1989 (see [10]); Bloor and Wilson proposed PDE method in 1989 and 1990 (see [2], [3]); Bajaj and Ihm proposed Hermite interpolation method in 1992 (see [1]); Wu and Wang proposed Wu's method in 1994 (see [12]); Zhu and Jin proposed the generatrix method in 1998 (see [8]); Wu and Zhou in 1995 (see [11]); Hartmann in 1994 and 2001 (see [5], [6]); Rossignac and Requicha proposed rolling ball method in 1984 (see [9]), Chen et al.(see [4]) use piecewise algebraic surfaces to blend

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pipe surfaces, and so on. Hartmman([6]) introduced a method for constructing G^n-continuous transition surfaces between two given normal ringed surfaces based on a recent G^n-blending method for parametric curves. Here a ringed surface is a surface generated by sweeping a circle with non constant radius along a curve. The ringed surface is called normal if the circle is contained in the normal planes of the curve. But the method is only fit for a special kind of surfaces. Chen et al.([4]) presented a scheme to blend three cylinders with piecewise cubic algebraic surfaces. They used 6 algebraic surfaces to form the whole blending algebraic surface with degree three. But to get one part of the blending surface, one needs fussy computation. And it is not easy to get the range of the parameters of the blending surface we need. Zhu and Jin([8]) presented a method which was based on generatrix for blending round or elliptical tubes. The basic idea of it is to design a basic generatrix and then change the parameter of the generatrix to form the blending surface. Wu and Wang([12]) studied the blending problem of several quadrics by using Wu's method and gave some examples in transition of pipelines. In these examples, the method can be used to find all possible blending surfaces with given degree. However, one has to do complicated symbolic computation in this way. And one is not sure which surface is a "good " surface that can be used in practice.When drawing it on computer, one has to seek a parametric representation of the implicit algebraic surface. And the blending surfaces are difficult(or not) to be adjusted. The problem still exists in Wu and Zhou's method ([11]). However they reduced the problem of finding blending algebraic surfaces to one of solving a linear system by virtue of the properties of Gröbner bases.

In this paper, we mainly discuss the smoothly joint problem of two quadric surfaces and derive the corresponding explicit formula. And we point out that the method can be expanded to n-way blending problems. The method is called the base curve method. It works as follows. We first construct a curve connecting the two axes of the surfaces to be blended. Based on the curve, we construct a collection of curves. And the blending surface is defined by these curves. Examples are given in this paper shown that this method gives nice solution to the problem. To get the blending surfaces, we only need the normal vector of the given surfaces on each point of their boundaries. Here the boundary curves are regular and continuous, and the normal vector at each point of the boundaries are well-defined and continuous. That means the blending surfaces are only defined by the boundary conditions. This is an extrusive advantage of the method. Further more, we can adjust the shape of the blending surface by adjust the base curve. And the method is easy to expand to solve other blending problems. And the blending surfaces having a parametric representation make them easy to be realized on computers or industrial applications. But the blending surfaces are non-rational.

2 The blending surface of quadric surfaces

Definition 1 Two C^1 continuous surfaces meet along a common boundary. Say they connect with G^1 ***continuity*** (or tangent plane continuity) if they have the same tangent plane at each point of the boundary and the unit normal vector is continuous along the common boundary.

Definition 2 Let C, S denote a curve and a surface respectively. $C \subset S$. The curvature of C at $P \subset C$ is called ***normal curvature*** of S at P if the unit main normal vector of C at P is same to the unit normal vector of s at P.

Definition 3 Two C^2 continuous surfaces meet along a common boundary. Say they connect with ***G^2 continuity*** if they are G^1 continuous along the common boundary , further more, the normal curvature on the point of boundary is continuous along the common boundary and is linear to the normal vector of the surfaces on the same point.

A space curve (surface(parametric or implicit). Here we only consider the parametric curve.) is called ***regular*** if the tangent vector (normal vector) at every point on the curve (surface) exists and is unique and nonzero. For example, if space curve ($P(t)$ = $(x(t), y(t), z(t)), t \in [0, 1]$) has a tangent vector $Q(t)$ =$(\frac{\partial x(t)}{\partial t}, \frac{\partial y(t)}{\partial t}, \frac{\partial z(t)}{\partial t})$ and $Q(t) \neq (0, 0, 0)$ for all t in $[0, 1]$, $P(t)$ is a regular curve. A regular space curve is called a base curve of a curve or a surface (***base curve*** for short) if the curve or the surface is constructed through the space curve based on some rule you give. That is, for example, we can regard X-axes as the base curve of the surface of revolution $(t, t^2 \cos\theta, t^2 \sin\theta)(t \in [1, 4], \theta \in [0, 2\pi])$ and the rule is revolving $y = t^2$ along X-axes.

Theorem. Let S_1, S_2 be regular surfaces, $C = C(t)$ be regular space curve, N = N(t) is the normal vector of S_1 at each point on C, $S_1 \bigcap S_2 = C$. $\forall\ P = C(t_0)$ ∈C, if there exists a regular space curve $C_2 = C_2(s) \subset S_2$ and $C_2(s_0) = P$, Further more,

$$\frac{C'(t_0) \times C_2'(s_0)}{\|C'(t_0) \times C_2'(s_0)\|} = \pm \frac{N(t_0)}{\|N(t_0)\|}. \tag{2.1}$$

Here $C'(t_0), C_2'(s_0)$ are the tangential vectors of the curves C, C_2 at point P respectively. Then S_1 and S_2 meet with G^1 continuity along C. $C'(t_0)$ is the first derivative of $C(t)$ when $t = t_0$. Here it denote the tangent line of curve $C(t)$ at $C(t_0)$ (Other similar denotations are the same meaning.).

Proof. The tangent plane of S_1 at P is $\{Q_1|(Q_1 - P) \cdot N(t_0) = 0\}$. And the tangent plane of S_2 at P is $\{Q_2|(Q_2 - P) \cdot (C'(t_0) \times C_2'(s_0)) = 0\}$. The two planes are obvious the same plane when (2.1) holds. As is shown in Figure 1. It means that the two surfaces are tangent plane continuity. So the theorem holds.

Corollary. Let S_1, S_2 be regular surfaces, $C = C(t)$ be regular space curves, $S_1 \bigcap S_2 = C$. $\forall\ P = C(t_0) \in C$, if there exists regular space curves $C_1 = C_1(s) \subset S_1$,$C_2 = C_1(s') \subset S_2$ and $C_1(s_0) = P$, $C_2(s_0') = P$, Further more $\exists\, a \in \mathbf{R}/\{0\}, \forall\, b \in \mathbf{R}$, such that

$$\begin{cases} C_1'(s_0) = a * C_2'(s_0') \\ C'(t_0) \neq b * C_1'(s_0) \end{cases} \tag{2.2}$$

Then S_1 and S_2 meet with G^1 continuity along C.

Make use of the definition of geometric continuity, the theorem and the corollary tell us a constructive method to construct blending surfaces. We will show you how to do so.

A quadric surface is a surface defined by a polynomial with degree two. And here we don't discuss the surfaces or other graphs defined by quadric polynomial such as $x^2 = \pm a^2, x^2 + b^2y^2 = 0$. And we

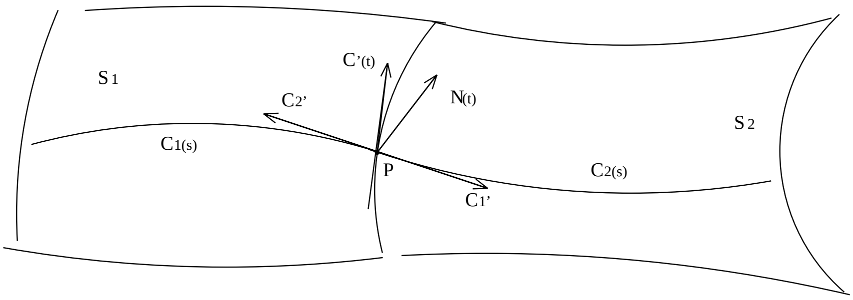

Figure 1: proof of the theory and corollary

mainly discussed the ***close*** surfaces, whose plane sections are either ellipses or circles if the planes intersect with the surfaces appropriately. In other words, they are 6 surfaces given below: elliptic cylinder, elliptic cone, elliptic paraboloid, elliptic sphere, hyperboloid of one sheet and hyperboloid of two sheet. There are 3 quadric surfaces which are not close. They are hyperbolic paraboloid, hyperbolic cylinder, parabolic cylinder.

Problem. Let S_1, S_2 be regular quadric surfaces, h_1, h_2 be two planes perpendicular to the axes of the surfaces respectively. We need to construct a blending surface which will intersect S_1 and S_2 along the intersection curves $S_1 \bigcap h_1$, $S_2 \bigcap h_2$ with G^1 continuity.

In what below, we show how to construct the blending surface.

2.1 Constructing the base curve

Let A, B be the points of intersection between the axes of quadric surfaces S_1, S_2 and h_1, h_2 respectively. Here the axes of a quadric close surface is a straight line enclosed by the surface. That is, for example, X-axes is the axes of surface $y^2 + z^2 = r^2$. The vertical distance and the angle between two axes are d_0 and α. Here the base curve is the curve connecting the two axes at A, B with G^1 continuity. The first step is to construct the base curve. We can consider one of the axes as the X-axis, the common vertical line of two axes as Z-axis. The other axes intersects Z-axis at point O'. O is the origin. And the Y-axis is vertical to both the X-axis and Z-axis. B is on the X-axis. Let $\overline{O'A} = d_1$, $\overline{OB} = d_2$. We can use many methods to construct the base curve. For example, B$\acute{e}$zier curve, Hermite interpolation and so on. Here we use B$\acute{e}$zier's method. We use A, B and other two points A_1, B_1 as the control points to construct the base curve. A_1 is on the same axes as A, and B_1 is on the same axes as B. Such that $\overline{OB_1} = l_2 d_2$, $\overline{AA_1} = (1 - l_1) d_1$. Here $l_1 \in (0, 1)$, $l_2 \in (0, 1)$. As it is shown in Figure 2, and curve $R_{t=0}\widetilde{P_t}R_{t=1}$ is the base curve. As we know, the base curve is G^1-contact with the two axes of the surfaces at points A and B respectively.

Then we can get the base curve defined by the following equation.

$$P(t) = A \cdot B_{3,0}(t) + A_1 \cdot B_{3,1}(t) + B_1 \cdot B_{3,2}(t) + B \cdot B_{3,3}(t) \tag{2.3}$$

In fact, a base curve is not needed always be B$\acute{e}$zier curve with 4 control points for our problem. We can use arc of an ellipse to contact the two axes when $d_0 = 0$. For the sake of avoiding the blending surface intersecting itself, the radius of curvature of the base curve at every point should be larger than the maximal

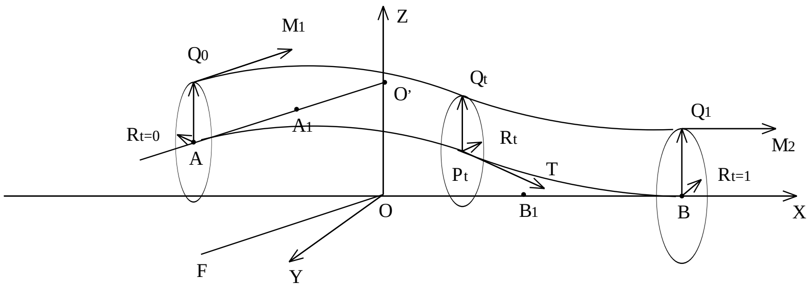

Figure 2: position in coordinate system

radius (in the normal plane of the base curve) at the point, which means that the following inequality should hold:

$$\frac{(\frac{(\partial x(t)}{\partial t}, \frac{\partial y(t)}{\partial t}, \frac{\partial z(t)}{\partial t}) \times (\frac{(\partial^2 x(t)}{\partial t^2}, \frac{\partial^2 y(t)}{\partial t^2}, \frac{\partial^2 z(t)}{\partial t^2})}{(\frac{\partial^2 x(t)}{\partial t^2} + \frac{\partial^2 y(t)}{\partial t^2} + \frac{\partial^2 z(t)}{\partial t^2})^{\frac{3}{2}}} \geq \max_{\theta\in[0,2\pi)} r(\theta, t), \forall t \in [0, 1] \tag{2.4}$$

Here $r(\theta, t)$ is defined by (2.15). We can adjust the value of t_1 and t_2 to satisfy this inequality for all t in [0,1]. Changing the value of t_1 and t_2 also can be used to rectify the shape of the base curve. That means the shape of the blending surface can be adjusted.

2.2 Designing the radius function

Now we have a base curve $(x(t), y(t), z(t)), t \in [0, 1]$. The second step is to construct the radius function. We will define a radius function: $r(\theta, t)$, $\theta \in [0, 2\pi)$, $t \in [0, 1]$. In the normal plane of the base curve at every point $P_t(x(t), y(t), z(t))$, there exists a ont-to-one correspondence between the real number in [0,2π) and the radials from P_t. Let h_0 be the normal plane of the base curve at P_t, R_t be the radial from P_t in h_0 which is parallel to the XOY-plane, R_θ be the radial from P_t in h_0 which forms an angle θ with R_t, Q_t be the intersection of R_θ and the blending plane to be constructed. Then $r(\theta, t)$ is the distance from P_t to Q_t. Obversely, it should be positive. To the same θ, we can define a regular continuous space curve by Q_t when t changes from 0 to 1. Let $S_\theta(t)$(It is curve $Q_0\widetilde{Q_t}Q_1$ as shown in figure 2) denote the curve. In order to connect the given surfaces smoothly, the tangential line of the curve at the extreme points should be in the tangential plane of the given surfaces. As is shown in the theorem. Let θ change in $[0, 2\pi)$. We can get a collection of curves. All these curves form the blending surface. Each point on the intersection curve ($S_1 \bigcap h_1$ or $S_2 \bigcap h_2$) and the axes of the given surface ($S_1 or S_2$) defines a plane, which intersects the given surface to a planar curve $C_{s1}(s)$(or $C_{s2}(s)$)(Here $C'_{s1}(s)(C'_{s2}(s))$ is equal to $M_1(M_2)$ at point $Q_0(Q_1)$). The tangent of angle between the tangential line of the curve at the point and the axes is $\tan\alpha_1$(or $\tan\alpha_2$), where α_i is a function of θ. We can use the planar vector $\overrightarrow{M'_i} = (1, M'_i(\theta)) = (1, \tan\alpha_i)$(i=1,2) to denote it (if the tangential line is vertical to the axes, then $\overrightarrow{M'_i} = (0, \pm 1)$). We show the radius function in Figure 2. Here $r_1(\theta)$ (or $r_2(\theta)$) is the distance from A (or B) to the point on the intersection curve which corresponds to $\theta \in [0, 2\pi)$. Form what we state above, we know $C_{s1}(s)$(or $C_{s2}(s)$) connecting $S_\theta(t)$ with G^1 continuity. When θ changes from 0 to 2π, $S_\theta(t)$ forms the blending surface $S(\theta, t)$. The corollary ensures that $S(\theta, t)$ connects S_1(or S_2) with G^1 continuity. We can get the following representation about the radius function.

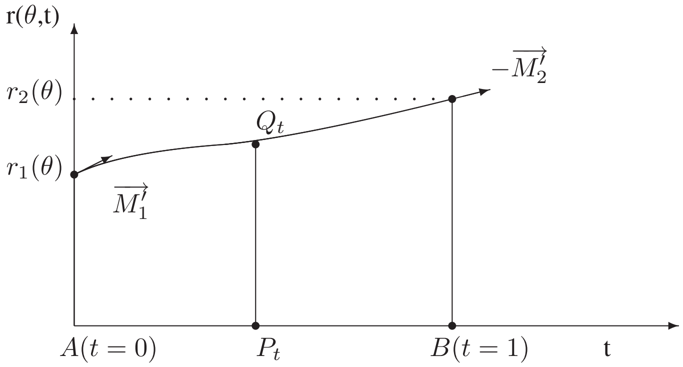

Figure 3: radius function(Q_t is a point of the blending curve.)

$$r(\theta, t) = \overline{P_t Q_t}, t \in [0,1], \theta \in [0, 2\pi) \tag{2.5}$$

We will show how to calculate the radius function of an elliptic cone and a cylinder for example. The standard functions of them are (It is not denoted the corresponding position in the coordinate system) :

$$\frac{y^2}{a^2} + \frac{z^2}{b^2} - c * x^2 = 0 \tag{2.6}$$

$$y^2 + z^2 - 0.25 = 0 \tag{2.7}$$

In order to simplify the calculation, we consider the parametric form of the surface (2.6).

$$\begin{cases} x(\theta, t) = t \\ y(\theta, t) = a \cos\theta f(t) \\ z(\theta, t) = b \sin\theta f(t) \end{cases} \tag{2.8}$$

Here $f(t) = \sqrt{c}\, t, t > 0$. Then

$$r_1(\theta) = f(d_1)\sqrt{a^2 \sin^2\theta + b^2 \cos^2\theta} \tag{2.9}$$

$$M_1(\theta) = \sqrt{a^2 \sin^2\theta + b^2 \cos^2\theta}\frac{\partial f(t)}{\partial t}|_{t=d_1} \tag{2.10}$$

So

$$r_1(\theta) = \sqrt{c} d_1 \sqrt{a^2 \sin^2\theta + b^2 \cos^2\theta} \tag{2.11}$$

$$M_1(\theta) = \sqrt{c}\sqrt{a^2 \sin^2\theta + b^2 \cos^2\theta} \tag{2.12}$$

In the same way, we can get the following result.

$$r_2(\theta) = 0.5 \tag{2.13}$$

$$M_2(\theta) = 0 \tag{2.14}$$

The curve defined by the radius function to be constructed should pass $P_0(0, r_1(\theta))$, $P_1(1, r_2(\theta))$, and the tangent vectors of which at P_0, P_1 are the same with $\overrightarrow{M_1}$, $\overrightarrow{M_2}$ respectively. We can use Hermite interpolation method to get the radius function:

$$r(\theta, t) = (M_1 - M_2 - 2r_2 + 2r_1)t^3 + (3r_2 - 3r_1 - 2M_1 + M_2)t^2 + M_1 t + r_1 \tag{2.15}$$

Here t($\in [0, 1]$) increases along the base curve from A to B. We also can construct a B$\acute{e}$zier curve to get the radius function.

2.3 Getting the parametric blending surface

From the discussion above,we can get the expression of the blending surface. One can prove that it connects the given surfaces with tangent plane continuity.

$$\begin{cases} x(\theta,t) = r(\theta,t)(\frac{-\frac{\partial x(t)}{\partial t}\frac{\partial z(t)}{\partial t}}{\sqrt{((\frac{\partial x(t)}{\partial t})^2+(\frac{\partial y(t)}{\partial t})^2+(\frac{\partial z(t)}{\partial t})^2)((\frac{\partial x(t)}{\partial t})^2+(\frac{\partial y(t)}{\partial t})^2)}}\sin\theta \\ \qquad + \frac{-\frac{\partial y(t)}{\partial t}}{\sqrt{(\frac{\partial x(t)}{\partial t})^2+(\frac{\partial y(t)}{\partial t})^2}}\cos\theta) + x(t) \\ y(\theta,t) = r(\theta,t)(\frac{-\frac{\partial y(t)}{\partial t}\frac{\partial z(t)}{\partial t}}{\sqrt{((\frac{\partial x(t)}{\partial t})^2+(\frac{\partial y(t)}{\partial t})^2+(\frac{\partial z(t)}{\partial t})^2)((\frac{\partial x(t)}{\partial t})^2+(\frac{\partial y(t)}{\partial t})^2)}}\sin\theta \\ \qquad + \frac{\frac{\partial x(t)}{\partial t}}{\sqrt{(\frac{\partial x(t)}{\partial t})^2+(\frac{\partial y(t)}{\partial t})^2}}\cos\theta) + y(t) \\ z(\theta,t) = r(\theta,t)\frac{(\frac{\partial x(t)}{\partial t})^2+(\frac{\partial y(t)}{\partial t})^2}{\sqrt{((\frac{\partial x(t)}{\partial t})^2+(\frac{\partial y(t)}{\partial t})^2+(\frac{\partial z(t)}{\partial t})^2)((\frac{\partial x(t)}{\partial t})^2+(\frac{\partial y(t)}{\partial t})^2)}}\sin\theta + z(t) \end{cases} \tag{2.16}$$

Here (x(t), y(t), z(t))is the parametric expression of the base curve. It can be defined by (2.3).

This method is easy to solve the blending problems of so-called normal ringed surfaces mentioned in [6]. We only need to change $r(\theta, t)$ to $r(t)$ in (2.16). $r(t)$ can still be defined by (2.15), but here M_i and r_i are constant to the given blending problem.

3 Examples

Example 1 In this example, we consider the connecting of an elliptic cylinder and an elliptic paraboloid. The standard equations are:

$$\frac{y^2}{a_2^2} + \frac{z^2}{b_2^2} - c_2 * x = 0 \tag{2.17}$$

$$\frac{y^2}{a_1^2} + \frac{z^2}{b_1^2} - 1 = 0 \tag{2.18}$$

We can use (2.3) to construct the base curve. The radius function can be defined by (2.15). Here

$$r_2(\theta) = \sqrt{c_2 d_2}\sqrt{a^2\sin^2\theta + b^2\cos^2\theta} \tag{2.19}$$

$$M_2(\theta) = \sqrt{a_2^2 \sin^2\theta + b_2^2 \cos^2\theta}\frac{c}{2\sqrt{cd_2}} \tag{2.20}$$

$$r_1(\theta) = \sqrt{a_1^2 \sin^2\theta + b_1^2 \cos^2\theta} \tag{2.21}$$

$$M_1(\theta) = 0 \tag{2.22}$$

Then we can get the blending surface defined by (2.16). And the parameters of the blending surface shown in Figure 4 are given below: $t_1 = t_2 = 0.5$, $d_0 = 0.3$, $d_1 = 0.5$, $d_2 = 0.6$, $\alpha = 5\pi/6$, $a_1 = 0.25$, $b_1 = 0.3$, $a_2 = 0.3$, $b_2 = 0.35$, $c_2 = 0.3$.

Example 2 Let us assume that the surfaces to be blended are two cylinders with intersecting axes. The two axes form an angle α. The radii of the cylinders are r_1 and r_2 respectively. Using the method introduced in section 2, we construct the blending surface. First, we can construct the base curve of the following form:

$$\begin{cases} x(t) = (d_1\cos\alpha + d_2)t^2 - 2d_1\cos\alpha t + d_1\cos\alpha \\ y(t) = d_1\sin\alpha t^2 - 2d_1\sin\alpha t + d_1\sin\alpha \\ z(t) = 0 \end{cases} \tag{2.23}$$

Secondly, we can get the following radius function by (2.15):

$$r(\theta, t) = 2(r_1 - r_2)t^3 - 3(r_1 - r_2)t^2 + r_1 \tag{2.24}$$

Then we can get the blending surface as the following form by (2.16):

$$\begin{cases} x(\theta, t) = r(\theta, t)(\frac{-\frac{\partial y(t)}{\partial t}}{\sqrt{(\frac{\partial x(t)}{\partial t})^2 + (\frac{\partial y(t)}{\partial t})^2}}\cos\theta) + x(t) \\ y(\theta, t) = r(\theta, t)(\frac{\frac{\partial x(t)}{\partial t}}{\sqrt{(\frac{\partial x(t)}{\partial t})^2 + (\frac{\partial y(t)}{\partial t})^2}}\cos\theta) + y(t) \\ z(\theta, t) = r(\theta, t)\sin\theta + z(t) \end{cases} \tag{2.25}$$

In order to get a "good" blending surface, we should think about inequality (2.4). The problem is transformed to the following form:

$$\frac{1}{2\sqrt{d_1^2 + d_2^2 + 2d_1d_2\cos\alpha}} \geq 2(r_1 - r_2)t^3 - 3(r_1 - r_2)t^2 + r_1, \forall\, t \in [0, 1] \tag{2.26}$$

One can easily get an equivalent representation of (2.26) as the following:

$$\frac{1}{2\sqrt{d_1^2 + d_2^2 + 2d_1d_2\cos\alpha}} \geq max\{r_1, r_2\} \tag{2.27}$$

The parameters of the graph shown in Figure 5 are : $r_1 = 0.2$, $r_2 = 0.3$, $d_1 = 0.4$, $d_2 = 0.3$, $\alpha = 5\pi/6$.

Example 3 In this example, we will show that our method introduced in section 2 can be modified to construct blending surfaces for non-planar cutting curves. Let us think about the blending of two cylinders

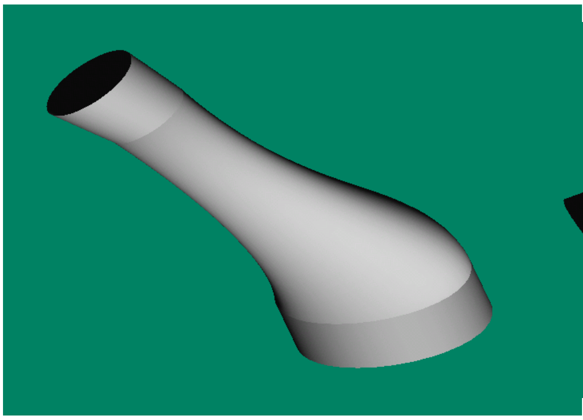

Figure 4: elliptic cylinder jointing with elliptic paraboloid

Figure 5: connecting of two cylinders

intersecting axes. The axes of two cylinders are vertical. One cylinder cuts in another. The equations of the cylinders in the coordinate system are given below.

$$y^2 + z^2 - r_1^2 = 0 \tag{2.28}$$

$$x^2 + z^2 - r_2^2 = 0 \tag{2.29}$$

Another cylinder is defined by the following equation.

$$y^2 + z^2 - r^2 = 0 \tag{2.30}$$

Where $r < r_2$. Cylinder (2.30) intersects cylinder (2.29) at a space curve. We can get the parametric equation of it.

$$\begin{cases} x(\theta) = \sqrt{r_2^2 - r^2 sin^2\theta} \\ y(\theta) = r\cos\theta \\ z(\theta) = r\sin\theta \end{cases} \tag{2.31}$$

We can get the blending surface with the following representation.

$$\begin{cases} x(\theta, t) = d_0 - t \\ y(\theta, t) = r(t)\cos\theta \\ z(\theta, t) = r(t)\sin\theta \end{cases} \tag{2.32}$$

Here $t \in [0, t(\theta)]$. And $t(\theta)$, r(t) are defined by the equations below:

$$t(\theta) = d_0 - \sqrt{r_2^2 - r^2\sin^2\theta} \tag{2.33}$$

.

$$r(t)=(2r_1-2r+3l_2\sqrt{\frac{r_2^2-r^2\sin^2\theta}{r_2^2-r^2\sin^2\theta\cos^2\theta}})t^3+(-3r_1+3r-3l_2\sqrt{\frac{r_2^2-r^2\sin^2\theta}{r_2^2-r^2\sin^2\theta\cos^2\theta}})t^2+r_1 \quad (2.34)$$

Here the parameters l_1, l_2 are used to adjust the shape the radius function. That is, to adjust the shape of the blending surface. The parameters of the figure shown in Figure 6 are the following: $d_0 = 0.6$, $r_1 = 0.2$, $r_2 = 0.3$, $r = 0.2$, $l_1 = 0.1$, $l_2 = 0.1$.

Example 4 In this example, we will show several cylinders whose axes jointing at the same point connecting a sphere with G^1 continuity. We can use this method to solve n-way blending problems. The picture shown in Figure 7 is defined by the following parameters. The radius of the sphere is 0.3. The radiuses of the given cylinders are 0.15. The radius of the circle defined by a plane intersecting sphere is 0.2. The distances from the origin to the planes intersecting the given cylinders are $\sqrt{0.3^2 - 0.15^2} + 0.15$.

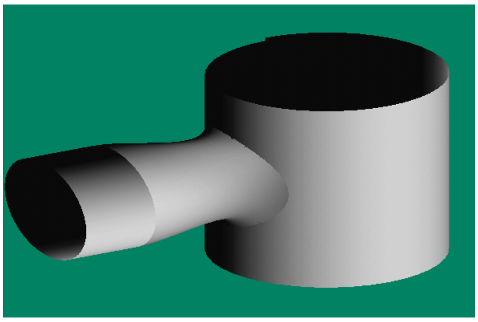

Figure 6: one cylinder inserting another

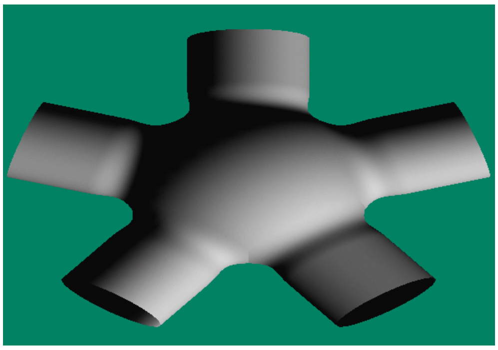

Figure 7: five cylinders jointing

4 Conclusion

A method for connecting two surfaces G^1-continuously is introduced. It is based on a G^1-continuous parametric regular curve. Obviously, this method can be extended to connecting general regular surfaces.

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