Hybrid Symbolic-Numeric Computation*

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TUTORIAL ABSTRACT

Several standard problems in symbolic computation, such as greatest common divisor and factorization of polynomials, sparse interpolation, or computing solutions to overdetermined systems of polynomial equations have non-trivial solutions only if the input coefficients satisfy certain algebraic constraints. Errors in the coefficients due to floating point round-off or through phsical measurement thus render the exact symbolic algorithms unusable. By symbolic-numeric methods one computes minimal deformations of the coefficients that yield non-trivial results. We will present hybrid algorithms and benchmark computations based on Gauss-Newton optimization, singular value decomposition (SVD) and structure-preserving total least squares (STLS) fitting for several of the above problems.

A significant body of results to solve those "approximate computer algebra" problems has been discovered in the past 10 years. In the Computer Algebra Handbook the section on "Hybrid Methods" concludes as follows [2]: "The challenge of hybrid symbolic-numeric algorithms is to explore the effects of imprecision, discontinuity, and algorithmic complexity by applying mathematical optimization, perturbation theory, and inexact arithmetic and other tools in order to solve mathematical problems that today are not solvable by numerical or symbolic methods alone." The focus of our tutorial is on how to formulate several approximate symbolic computation problems as numerical problems in linear algebra and optimization and on software that realizes their solutions.

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Approximate Greatest Common Divisors [3]. Our paper at this conference presents a solution to the approximate GCD problem for several multivariate polynomials with real or complex coefficients. In addition, the coefficients of the minimally deformed input coefficients can be linearly constrained. In our tutorial we will give a precise definition of the approximate polynomial GCD problem and we will present techniques based on parametric optimization (slow) and STLS or Gauss/Newton iteration (fast) for its numerical solution. The fast methods can compute globally optimal solutions, but they cannot verify global optimality. We show how to apply the constrained approximate GCD problem to computing the nearest singular polynomial with a root of multiplicity at least $k \geq 2$.

Approximate Factorization of Multivariate Polynomials [1]. Our solution and implementation of the approximate factorization problem follows our approach for the approximate GCD problem. Our algorithms are based on a generalization of the differential forms introduced by W. Ruppert and S. Gao to many variables, and use SVD or STLS and Gauss/Newton optimization to numerically compute the approximate multivariate factors.

Solutions of Zero-dimensional Polynomial Systems [4]. We translate a system of polynomials into a system of linear partial differential equations (PDEs) with constant coefficients. The PDEs are brought to an involutive form by symbolic prolongations and numeric projections via SVD. The solutions of the polynomial system are obtained by solving an eigen-problem constructed from the null spaces of the involutive system and its geometric projections.

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