Several Applications of the Characteristic Set Method

Kai Huang, Dingkang Wang & Lihong Zhi
Institute of Systems Science, Academia Sinica
Beijing 100080

In this paper we introduce the basic concepts of Characteristic Set method which considers the zero structure of polynomial sets . Mechanical theorem proving in geometry is a classic subject of artificial intelligence. As an application of characteristic set method, it consists of the following steps.

Step 1. Compute a characteristic set CS of HYP, if CS contains a constant the hypotheses of the theorem is self contradictory and stop.

Step 2 compute the remainder $R = prem(C, CS)$ if $R = 0$, then the theorem is true under the subsidiary conditions $I_1, \cdots, I_r \neq 0$. $I_1, \cdots, I_r$ are the initial set of CS. Otherwise if CS is irreducible, then the theorem is false under the condition $I_i \neq 0$, in both cases proceed next to step 4.

Step 3. if $R \neq 0$ and CS is reducible, then decompose HYP and determinate whether the theorem is true for each component.

Step 4. Analyze whether the subsidiary conditions are non-degeneracy conditions.

Mechanical derivation of unknown relations and locus equations, solving inverse kinematic equations of PUMA-type robots are also discussed.

We also give example of soving the following CAGD problems.

Problem Given in real 3-space R^3 three sets of irreducible algebraic curves C_i, C_j, C_k with $i \in I, j \in J, k \in K$ respectively, I, J, K being all finite sets of indices. Given also two sets of irreducible algebraic surfaces $S_j, S_k, (j \in J, k \in K)$ containing C_j, C_k respectively. To determine an irreducible algebraic surface S of given degree m verifying the following conditions:

(a) S contains all the curves C_i, C_j, C_k, for $i \in I, j \in J, k \in K$.

(b) S touches smoothly each of S_j, S_k along the curves C_j, C_k respectively, for $j \in J, k \in K$. More precisely, for each point on C_j or C_k which is regular for C_j, S, S_j or for C_k, S, S_k, S and S_j or S and S_k have same tangent planes at that point.