ACA 2022

Applications of Computer Algebra

D-finite Functions and Beyond:

Algorithms, Combinatorics, and Arithmetic

Program

August 15-19, 2022
Istanbul-Gebze, Turkey

Timetable of ACA2022

<table>
<tr><th>Times</th><th colspan="3">MON – 08/15</th><th colspan="3">TUE – 08/16</th><th colspan="3">WED – 08/17</th><th colspan="3">THU – 08/18</th><th colspan="3">FRI – 08/19</th><th>Times</th></tr>
<tr><td>9:30-10:00</td><td colspan="3">OPENING</td><td rowspan="3">S4</td><td rowspan="3">S5</td><td rowspan="3">S8</td><td rowspan="3">S1</td><td rowspan="3">S5</td><td rowspan="3">S8</td><td rowspan="2">S1</td><td rowspan="3">S6</td><td rowspan="3">S11</td><td rowspan="3">S1</td><td rowspan="3">S6</td><td rowspan="3">S3</td><td>9:30-10:00</td></tr>
<tr><td>10:00-10:30</td><td rowspan="2">S4</td><td rowspan="2">S14</td><td rowspan="2">S5</td><td>10:00-10:30</td></tr>
<tr><td>10:30-11:00</td><td></td><td>10:30-11:00</td></tr>
<tr><td>coffee</td><td colspan="3"></td><td colspan="3"></td><td colspan="3"></td><td colspan="3"></td><td colspan="3"></td><td>CB</td></tr>
<tr><td>11:30-12:00</td><td rowspan="2">S4</td><td rowspan="2">S2</td><td rowspan="2">S9</td><td colspan="3" rowspan="2">Invited Talk</td><td rowspan="2">S1</td><td>S2</td><td rowspan="2">S8</td><td colspan="3" rowspan="2">Invited Talk</td><td rowspan="2">S2</td><td rowspan="2">S6</td><td rowspan="2">S15</td><td>11:30-12:00</td></tr>
<tr><td>12:00-12:30</td><td></td><td>12:00-12:30</td></tr>
<tr><td>lunch</td><td colspan="3"></td><td colspan="3"></td><td colspan="3"></td><td colspan="3"></td><td colspan="3"></td><td>LB</td></tr>
<tr><td>14:00-14:30</td><td rowspan="4">S4</td><td rowspan="4">S5</td><td rowspan="4">S14</td><td rowspan="4">S9</td><td rowspan="4">S5</td><td rowspan="4">S11</td><td rowspan="4">S3</td><td rowspan="3">S13</td><td rowspan="4">S8</td><td colspan="3" rowspan="9">Excursion</td><td rowspan="2">S3</td><td rowspan="2">S6</td><td>S15</td><td>14:00-14:30</td></tr>
<tr><td>14:30-15:00</td><td></td><td>14:30-15:00</td></tr>
<tr><td>15:00-15:30</td><td colspan="3">CLOSING</td><td>15:00-15:30</td></tr>
<tr><td>15:30-16:00</td><td>S5</td><td colspan="3" rowspan="6"></td><td>15:30-16:00</td></tr>
<tr><td>coffee</td><td colspan="3"></td><td colspan="3"></td><td colspan="3"></td><td>CB</td></tr>
<tr><td>16:30-17:00</td><td rowspan="4">S4</td><td rowspan="4">S13</td><td rowspan="4"></td><td rowspan="3">S6</td><td rowspan="4">S13</td><td rowspan="4">S11</td><td rowspan="2">S8</td><td rowspan="2">S5</td><td rowspan="2"></td><td>16:30-17:00</td></tr>
<tr><td>17:00-17:30</td><td>17:00-17:30</td></tr>
<tr><td>17:30-18:00</td><td colspan="3">Group Picture</td><td>17:30-18:00</td></tr>
<tr><td>18:00-18:30</td><td></td><td colspan="3">ACA BM</td><td>18:00-18:30</td></tr>
</table>

S1	Computational Differential and Difference Algebra and its Applications
S2	Computer Algebra in Education
S3	Computer Algebra Modeling in Science and Engineering
S4	Effective ideal theory and combinatorial techniques in commutative and non commutative rings and their applications
S5	Algorithmic and Experimental Combinatorics
S6	D-finite Functions and Beyond: Algorithms, Combinatorics, and Arithmetic
S8	Algebraic and Geometric Methods in Coding Theory
S9	Parametric Polynomial Systems
S11	q-analogues in combinatorics: matroids, designs and codes
S13	Computer Algebra Applications in the life sciences
S14	Algorithms in Cryptography and Blockchain
S15	General Session

Our session is S6 in the schedule of ACA2022 with 4 parts:

1. **August 16, 16:30 - 18:30 (3 talks)**
2. **August 18, 9:30 -11:00 (3 talks)**
3. **August 19, 9:30 -11:00 (3 talks)**
4. **August 19, 11:30 - 15:00 (4 talks)**

Program

August 16, 2022 (Tuesday) S6

Time	Title	Speaker
16:30-17:00	q-Difference Equation Systems for Cylindric Partitions	Ali Uncu
17:00-17:30	Series defined by quadratic differential equations	Bertrand Teguia Tabuguia
17:30-18:00	Symbolic-Numeric Factorization of Differential Operators	Alexandre Goyer

August 18, 2022 (Thursday) S6

Time	**Title**	**Speaker**
9:30-10:00	Shift equivalence testing of polynomials and symbolic summation of multivariate rational functions	Lixin Du
10:00-10:30	Arithmetic of polynomial dynamical systems	Mohammad Sadek
10:30-11:00	Decision Problems for Second-Order Holonomic Recurrences	Eike Neumann

August 19, 2022 (Friday) S6

Time	Title	Speaker
9:30-10:00	C^2 -finite Sequences: A Computational Approach	Philipp Nuspl
10:00-10:30	Factoring differential operators in positive characteristic	Raphael Pages
10:30-11:00	Working with DD-finite functions automatically on SageMath	Antonio Jiménez-Pastor

August 19, 2022 (Friday) S6

Time	Title	Speaker
11:30-12:00	Galois groups of linear difference-differential equations	Ruyong Feng
12:00-12:30	Computing logarithmic parts by evaluation homomorphisms	Ziming Li
Lunch Break		
14:00-14:30	Efficient q-integer linear decomposition of multivariate polynomials	Hui Huang
14:30-15:00	D-finiteness, rationality, and height	Jason P. Bell

Applications of Computer Algebra – ACA 2022
Gebze-Istanbul, Turkey, | August 15-19, 2021
Session on *"D-finite Functions and Beyond: Algorithms, Combinatorics, and Arithmetic"*

D-finiteness, rationality, and height

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We discuss the growth of heights of coefficients of a D-finite series, showing that under conditions that ensure sufficiently slow growth, a D-finite series is necessarily rational.

Keywords
Heights, Pólya-Carlson theorem, Growth, Gap theorems

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[1] J. BELL, K. NGUYEN, U. ZANNIER, D-finiteness, rationality, and height. *Trans. Amer. Math. Soc.* **373**(7), 4889–4906 (2020).
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Shift equivalence testing of polynomials and symbolic summation of multivariate rational functions

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The Shift Equivalence Testing (SET) of polynomials is deciding whether two polynomials $p(x_1, \ldots, x_n)$ and $q(x_1, \ldots, x_n)$ satisfy the relation $p(x_1+a_1, \ldots, x_n+a_n) = q(x_1, \ldots, x_n)$ for some $a_1, \ldots, a_n$ in the coefficient field. The SET problem is one of basic computational problems in computer algebra and algebraic complexity theory, which was reduced by Dvir, Oliverira and Shpilka in 2014 to the Polynomial Identity Testing (PIT) problem [1]. In this talk, we presents a general scheme for designing algorithms to solve the SET problem which includes Dvir-Oliverira-Shpilka's algorithm as a special case. With the algorithms for the SET problem over integers, we give complete solutions to two challenging problems in symbolic summation of multivariate rational functions, namely the rational summability problem and the existence problem of telescopers for multivariate rational functions. Our approach is based on the structure of isotropy groups of polynomials introduced by Sato in 1960s [2]. Our results can be used to detect the applicability of the Wilf-Zeilberger method to multivariate rational functions.

Keywords
Summability, Telescopers, Isotropy Groups, Shift Equivalences

References

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Galois groups of linear difference-differential equations

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We consider the following $\sigma\delta$-linear system

$$\begin{cases}\sigma(Y) = AY \\ \delta(Y) = BY\end{cases}, \ A \in \mathrm{GL}_n(k_0(x)), B \in \mathrm{gl}_n(k_0(x))$$

where A, B satisfy the integrability condition: $\sigma(B)A = \delta(A) + AB$. Here (k_0, δ) is a differential field with algebraically cosed $C = k_0^\delta$, $k_0(x)$ is a $\sigma\delta$-field with shift operator $\sigma(x) = x + 1$. With respect to the above system, there are three algebraic subgroups of $\mathrm{GL}_n(C)$: the $\sigma\delta$-Galois group G of the above system over $k_0(x)$, the σ-Galois group G_{σ,c_1} of $\sigma(Y) = A^{c_1}Y$ over $C(x)$, and the δ-Galois group G_{δ,c_2} of $\delta(Y) = B^{c_2}Y$ over k_0, where $A^{c_1} \in \mathrm{GL}_n(C(x))$ and $B^{c_2} \in \mathrm{gl}_n(k_0)$ are specializations of A and B respectively.

We show that both G_{σ,c_1} and G_{δ,c_2} are algebraic subgroups of G under certain conditions on c_1, c_2, and $G = G_{\sigma,c_1}G_{\delta,c_2}$ for suitable c_1, c_2. These results enable us to reduce the problem of determining $\sigma\delta$-Galois groups to the problems of determining σ-Galois groups and δ-Galois groups. We also give a criterion for testing linear dependence of elements in a simple $\sigma\delta$-ring, which generalizes the classic results for elements in a σ-field or a δ-field and a result for hypexponential elements given by Li et al. 2007.

Keywords
Linear difference-differential equations, Galois groups, Specializations

Symbolic-Numeric Factorization of Differential Operators

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I am going to present a symbolic-numeric Las Vegas algorithm for factoring Fuchsian ordinary differential operators with rational function coefficients. The new algorithm combines ideas of van Hoeij's "local-to-global" method and of the "analytic" approach proposed by van der Hoeven. It essentially reduces to the former in "easy" cases where the local-to-global method succeeds, and to an optimized variant of the latter in the "hardest" cases, while handling intermediate cases more efficiently than both.

Keywords
Linear differential equations, Monodromy, Rigorous Numerics

References
[1] F. CHYZAK; A. GOYER; M. MEZZAROBBA, Symbolic-Numeric Factorization of Differential Operators. In *Proceedings of the 2022 International Symposium on Symbolic and Algebraic Computation*. `https://www.issac-conference.org/2022/`. Not published yet.

Efficient q-integer linear decomposition of multivariate polynomials

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We present two new algorithms for the computation of the q-integer linear decomposition of a multivariate polynomial. Such a decomposition is essential for the treatment of q-hypergeometric symbolic summation via creative telescoping and also for describing the q-counterpart of Ore-Sato theory. Both of our algorithms require only basic integer and polynomial arithmetic and work for any unique factorization domain containing the ring of integers. Complete complexity analyses are conducted for both our algorithms and two previous algorithms in the case of multivariate integer polynomials, showing that our algorithms have better theoretical performances. A Maple implementation is also included which suggests that our algorithms are much faster in practice than previous algorithms.

Keywords
q-Analogue, Integer-linear polynomials, Polynomial decomposition, Newton polytope, Creative telescoping, Ore-Sato theory

Working with DD-finite functions automatically on SageMath

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In this talk we are going to present the SageMath [5] package `dd_functions` and its latest features concerning DD-finite functions.

DD-finite functions are a natural extension of the holonomic framework. Holonomic (or *D-finite*) functions are formal power series ($f(x) \in \mathbb{K}[[x]]$) that satisfy linear differential equations with polynomials coefficients. These functions form a computable differential ring, namely, the elements can be represented on the computer, and all the ring operations and the derivative can be automatically executed [4]. Hence, they can be use again as coefficients for new differential equations leading to the definition of DD-finite functions.

Definition. [DD-finite] Let $f(x) \in \mathbb{K}[[x]]$. We say that $f(x)$ is *DD-finite* if and only if there is a natural number $d > 0$ and D-finite functions $r_0(x), \ldots, r_d(x)$ ($r_d(x) \neq 0$) such that

$$r_d(x)f^{(d)}(x) + \ldots + r_0(x)f(x) = 0.$$

This definition allows representing DD-finite functions with a finite amount of data since we only need to store the coefficients of the defining differential equation and some initial values $f(0), f'(0), \ldots, f^{(r)}(0)$.

It was shown in [2] that the set of DD-finite functions is also a computable differential ring (as it happened with the D-finite case). In fact, we can extend these results to the case were the coefficients are in a computable differential ring.

Definition. [Differentially definable] Let $R \subset \mathbb{K}[[x]]$ be a differential subring and $f(x) \in \mathbb{K}[[x]]$. We say that $f(x)$ is *differentially definable over* R if there is $d > 0$ and $r_0, \ldots, r_d \in R$ (with $r_d \neq 0$) such that

$$r_d f^{(d)}(x) + \ldots + r_0 f(x) = 0.$$

Theorem [3]. Let $R \subset \mathbb{K}[[x]]$ be a differential subring and let $D(R)$ be the set of all differentially definable functions over R. Then $D(R) \subset \mathbb{K}[[x]]$ is a computable differential ring.

With this result, we can observe that the differentially definable construction can be iterated, obtaining a chain of computable differential rings within $\mathbb{K}[[x]]$:

$$R \subset D(R) \subset D^2(R) \subset \ldots \subset D^n(R) \subset \ldots,$$

and, in this context, is clear that DD-finite functions are $D^2(\mathbb{K}[x])$.

These results were implemented in the SageMath [5] package `dd_functions` that we present in this talk. This software allows to construct any differentially definable ring and manipulate symbolically their elements in an automatic fashion.

This software is publicly available on GitHub*, and it is constantly updated with the new results concerning DD-finite and differentially functions [1]. It includes:

Structures

- Definition of any differentially definable ring.
- The possibility of working in the chain of $D^n(R)$.
- Create any differentially definable function giving the coefficients for the differential equation and some initial conditions.
- Use an always increasing library of examples coming from special functions.

Operations

- All closure properties are included.
- Composition of differentially definable functions $f(g(x))$ when $g(0) = 0$.
- Computing closure properties keeping the singularities of the differential equations.

Keywords
d-finite; dd-finite; formal power series; SageMath; special functions

References

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[2] A. JIMÉNEZ-PASTOR, V. PILLWEIN, A computable extension for D-finite functions: DD-finite functions. *Journal of Symbolic Computations* **94**, 90–104 (2019).
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Computing Logarithmic Parts by Evaluation Homomorphisms

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*`https://www.github.com/Antonio-JP/dd_functions`

Let $(K, ')$ be a differential field of characteristic zero, t be transcendental over K and t' belong to $K[t]$. Assume that K and $K(t)$ have the same subfield C of constants. A polynomial p in $K[t]$ is said to be normal if $\gcd(p, p') = 1$. A rational function f in $K(t)$ is said to be simple if it is proper and has a normal denominator.

Let $f \in K(t)$ be simple. Then f has an elementary integral if and only if

$$\int f = c_1 \log g_1 + \cdots + c_m \log g_m$$

for some $c_1, \ldots, c_m$ in the algebraic closure of C and $g_1, \ldots, g_m$ in $K(c_1, \ldots, c_m)(t)$. We call $\{(c_1, g_1), \ldots, (c_m, g_m)\}$ a logarithmic part of f when the above equality holds.

Given a simple function f, known algorithms for determining its logarithmic parts are based on either resultants [1, 6, 7], or subresultants [1, 3, 4], or Gröbner bases [1, 2, 5]. These algorithms need to find a polynomial $r \in K[z]$, where z is a constant indeterminate and r is either the Rothstein-Trager resultant of f [1,6,7] or its squarefree part. Then f has a logarithmic part if and only if the monic associate p of r belongs to $C[z]$. It is time-consuming to compute r when K is a field of multivariate rational functions over C.

We present a new algorithm that computes a candidate $q \in C[z]$ for the monic associate p by evaluation homomorphisms, and attempts to construct a logarithmic part of f using q by algebraic gcd-computation. By a property of residue multiplicities, the algorithm either confirms the non-existence of logarithmic parts or finds a logarithmic part of f. Empirical results illustrate that the algorithm is more efficient than the known algorithms.

Keywords
Elementary integral, Evaluation homomorphism, Logarihtmic part, Simple function

References

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Decision Problems for Second-Order Holonomic Recurrences

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We study decision problems for sequences which obey a second-order holonomic recurrence of the form $f(n+2) = P(n)f(n+1) + Q(n)f(n)$ with rational polynomial coefficients, where P is non-constant, Q is non-zero, and the degree of Q is smaller than or equal to that of P. We show that existence of infinitely many zeroes is decidable. We give partial algorithms for deciding the existence of a zero, positivity of all sequence terms, and positivity of all but finitely many sequence terms. If Q does not have a positive integer zero then our algorithms halt on almost all initial values $(f(1), f(2))$ for the recurrence. We identify a class of recurrences for which our algorithms halt for all initial values. We further identify a class of recurrences for which our algorithms can be extended to total ones.

Keywords
Holonomic sequences, Positivity Problem, Skolem Problem

C^2-finite Sequences: A Computational Approach

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We define a class of sequences which satisfy a linear recurrence with coefficients that, in turn, satisfy a linear recurrence with constant coefficients themselves, i.e., are C-finite. These C^2-finite sequences are a natural generalization of P-finite sequences, they form a ring and satisfy additional computational properties [1,2,3]. It turns out that, compared to P-finite sequences, the algorithmic aspects are much more involved and are related to difficult problems in number theory. We give an introduction to these C^2-finite sequences and present an implementation in the computer algebra system SageMath.

Keywords
Difference equations, holonomic sequences, closure properties

References

[1] A. Jiménez-Pastor, P. Nuspl, V. Pillwein, On C^2-finite sequences. In *ISSAC'21*, F. Chyzak, G. Labahn (eds.), 217–224. 2021.
[2] A. Jiménez-Pastor, P. Nuspl, V. Pillwein, An extension of holonomic sequences: C^2-finite sequences. In *Journal of Symbolic Computation*. accepted. 2022.
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Factoring differential operators in positive characteristic

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We present an algorithm to factor differential operators with coefficients in an algebraic function field K of characteristic p, provided with the usual derivation, as a product of irreducible differential operators with coefficients in K. We make use of tools specific to the characteristic p, such as the p-curvature or the arising central simple algebra structure. In particular we shall see that factoring differential operators ultimately reduces to solving some "p-Ricatti" equations, for which purpose we use tools of algebraic geometry.

Keywords
Differential operators, Factorisation, Positive characteristic, p-curvature, Central simple algebras

Arithmetic of polynomial dynamical systems

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The number theoretic properties of iterations of polynomial maps defined over number fields are governed by the degree of the maps and the degree of the field. Although due attention has been given to iterations of quadratic polynomial maps over number fields of small degree, arithmetic dynamical systems produced by iterations of polynomial maps of higher degrees have not been addressed much in literature. In this talk, we survey some of the old and new results on arithmetic polynomial dynamical systems. The focus will be on algebraic aspects of these systems in the case that the degree of the polynomial map is at least three.

Keywords
Arithmetic dynamics, Dynamical irreducibility, Periodic points

Series defined by quadratic differential equations

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Differential polynomials of degree at most one annihilate D-finite functions. We consider annihilators of degree at most two and present a general strategy to represent power series solutions of resulting differential equations given enough initial values [1]. Using techniques

from algebraic geometry (see [2]), our method extends to representations of Laurent-Puiseux series. Consequently, we can prove identities beyond D-finiteness. However, doing so raises the question of closure properties. Indeed, to show equivalence between two expressions, we may need to establish evidence of the zero-equivalence of their difference; therefore, in a sense, it is relevant to know if the class under consideration contains additive group structures. We present some of our investigations around closure properties with generalization to differential polynomials of degree at most $k \in \mathbb{N}$.

Furthermore, we demonstrate how our method highlights a reverse methodology that finds application in Guessing: recovering a non-D-finite function from a truncation of its power series expansion [3].

Parts of this presentation came from joint work with Wolfram Koepf and Anna-Laura Sattelberger.

Keywords
Differential algebra, Power series representation, Guessing

References
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q-Difference Equation Systems for Cylindric Partitions

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[2] University of Bath, Department of Computer Science, Bath UK

The cylindric partitions defined by Gessel and Krattenthaler [4] attracted interest after a recent paper by Corteel and Welsh [3]. In this talk, we will look at these objects and their symmetric versions as well as skew double shifted plane partitions. We will especially focus on the coupled q-difference equation systems that these objects are associated with and the difficulties of solving such systems.

Parts of this work is joint with Sylvie Corteel, Jehanne Dousse [2], and Walter Bridges [1].

Keywords
Cylindric Partitions, q-Difference Equations, Computer Algebra

References
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