HypergeometricCT Package

```
> restart;
> read("HypergeometricCT.mm");
> with(HypergeometricCT);
```

$$[DispersionSet, HypergeomReduction, HypergeomTelescoping, OrderBounds, PolynomialReduction, RNF, ShellReduction, ShiftFactorization, ShiftQuotient, VerifyHypergeomReduction, VerifyHypergeomTelescoping, VerifyShellReduction] \quad \textbf{(1.1)}$$

HypergeomReduction

Calling sequence:

HypergeomReduction(T,y)

Input: T, a hypergeometric term over F(y);

y, a variable name.

Output: [[g,r],H], where

g,r - two rational functions in F(y),

H - a hypergeometric term over F(y) with Sy(H)/H = K being Sy-reduced,

such that

(1) T = (Sy-1)(g*H)+r*H;

(2) r is a Sy-remainder w.r.t. K.

```
> T := y^2*y!/(y+1);
```

$$T := \frac{y^2\, y!}{y+1} \quad \textbf{(2.1)}$$

```
> res := HypergeomReduction(T,y); #Apply the algorithm HypergeomReduction to T w.r.t. y
```

$$res := \left[\left[\frac{y}{y+1}, -\frac{1}{y+2}\right], y!\right] \quad \textbf{(2.2)}$$

```
> VerifyHypergeomReduction(res,T,y);
```

$$true \quad \textbf{(2.3)}$$

```
> SumTools[Hypergeometric][Gosper](T,y); #Apply Gosper's algorithm to T w.r.t. y
Error, (in SumTools:-Hypergeometric:-Gosper) no solution found
> SumTools[Hypergeometric][SumDecomposition](T,y); #Apply the Abramov-Petkovsek reduction to T w.r.t. y
```

$$\left[\frac{y\left(\prod_{\_k=1}^{y-1}(\_k+1)\right)}{y+1}, -\frac{\prod_{\_k=1}^{y-1}(\_k+1)}{y+2}\right] \quad \textbf{(2.4)}$$

HypergeomTelescoping

Calling sequence:

HypergeomTelescoping(T,x,y,Sx,'cert')

Input: T, a hypergeometric term over QQ(x,y);
x,y, two variable names;
Sx, an operator name used for the shift operator w.r.t. x;
cert, (optional) an unevaluated name used for storing the certificate.

Output: L, a nonzero linear difference operator in QQ(x)[Sx]
such that
L(T) = (Sy-1)(G),
where G is a hypergeometric term over QQ(x,y), if such an L exists.
The optional 5th argument 'cert' is assigned [g,H] with G = g*H,
where g is a rational function in QQ(x,y) and H is a hypergeometric
term over QQ(x,y) whose Sy-quotient is Sy-reduced.
"No telescoper exists!",
if there is no such L.

```
> T := binomial(x,y)^3;
```

$$T := \binom{x}{y}^3 \tag{3.1}$$

```
> L := HypergeomTelescoping(T,x,y,Sx,'cert'); #Apply the algorithm HypergeomTelescoping to T w.r.t. x and y, yielding a minimal telescoper and a corresponding certificate
cert;
```

$$L := -\frac{8\left(x^2+2x+1\right)}{x^2+4x+4} - \frac{\left(7x^2+21x+16\right)Sx}{x^2+4x+4} + Sx^2$$

$$\left[\frac{1}{(x+2)^2\,(x-y+1)^3\,(x+2-y)^3}\left(-14x^5y^3+27x^4y^4-18x^3y^5+4x^2y^6-102x^4y^3+147x^3y^4-66x^2y^5+8xy^6-290x^3y^3+291x^2y^4-78xy^5+4y^6-402x^2y^3+249xy^4-30y^5-272xy^3+78y^4-72y^3\right), \binom{x}{y}^3\right] \tag{3.2}$$

```
> VerifyHypergeomTelescoping([L,cert],T,x,y,Sx);
```

$$true \tag{3.3}$$

```
> HypergeomTelescoping(T,x,y,Sx); #Apply the algorithm HypergeomTelescoping to T w.r.t. x and y, yielding a minimal telescoper without computing a certificate
```

$$-\frac{8\left(x^2+2x+1\right)}{x^2+4x+4} - \frac{\left(7x^2+21x+16\right)Sx}{x^2+4x+4} + Sx^2 \tag{3.4}$$

```
> SumTools[Hypergeometric][Zeilberger](T,x,y,Sx); #Apply Zeilberger's algorithm to T w.r.t. x and y, yielding a minimal telescoper and a corresponding certificate
```

$$\left[\left(x^2+4x+4\right)Sx^2+\left(-7x^2-21x-16\right)Sx-8x^2-16x-8, \right. \tag{3.5}$$

$$\frac{1}{(x+2-y)^3\,(x-y+1)^3}\left(\left(y^3+\left(-\frac{9x}{2}-\frac{15}{2}\right)y^2+\left(\frac{27}{4}x^2+\frac{93}{4}x+\frac{39}{2}\right)y\right.\right.$$

$$- \frac{7 x^3}{2} - \frac{37 x^2}{2} - 32 x - 18\Big) y^3 \binom{x}{y}^3 \left(4 x^2 + 8 x + 4\right)\Big)\Big]$$