Appendix A The triangular sets in the zero decomposition for the P3P problem

$$\begin{cases} a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4, \\ b_0y - b_1. \end{cases} \tag{TS_1}$$

$$\begin{cases} a_5x^3 + a_6x^2 + a_7x + a_8, \\ b_2y - b_3, \\ a^2 + (-2 + 2b - br^2)a - 2b + b^2 + 1. \end{cases} \tag{TS_2}$$

$$\begin{cases} (r^2p^2 - 4pqr + r^2q^2)x^2 + (4p^2q - p^2r^2q)x - 4p^2 + r^2p^2, \\ b_4y - b_5, \\ (-4p^2 + 4pqr + r^2p^2 + r^2q^2 - r^3pq - 4q^2)a + r^2p^2 - 4pqr + 4q^2, \\ (-4p^2 + 4pqr + r^2p^2 + r^2q^2 - r^3pq - 4q^2)b + r^2q^2 + 4p^2 - 4pqr. \end{cases} \tag{TS_3}$$

$$\begin{cases} (p^2b + q^2b - p^2)x^2 + (-4bq + p^2q)x + 4b - p^2, \\ py + qx - 2, \\ a + b - 1, \\ r. \end{cases} \tag{TS_4}$$

$$\begin{cases} qx - 1, \\ py - 1, \\ (p^2 + q^2)a - q^2, \\ (p^2 + q^2)b - p^2, \\ r. \end{cases} \tag{TS_5}$$

$$\begin{cases} qx - 1, \\ py - 1, \\ (p^4 - 2p^2q^2 + q^4)a - p^2q^2 - q^4, \\ (p^4 - 2p^2q^2 + q^4)b - p^2q^2 - p^4, \\ (p^2 + q^2)r - 4pq. \end{cases} \tag{TS_6}$$

$$\begin{cases} (p^2q - 2pr)x - p^2 + r^2, \\ py - 1, \\ (4r^2 + p^2q^2 + p^4 - r^4 - p^3qr + pr^3q - 4qpr)b \\ +2pr^3q - 2p^2r^2 + 2p^3qr - p^2q^2r^2 - p^4 - r^4. \end{cases} \tag{TS_7}$$

$$\begin{cases} (2pr^3q - 2p^2r^2 + 4br^2 + p^2q^2b + p^4b - r^4b + 2p^3qr - p^2q^2r^2 - p^4 - r^4 - \\ p^3qrb + r^3pbq - 4qbpr)x^2 + (-q^2r^3pb + 2pq^2br + 2p^2r^2q + p^2q^3r^2 + r^4q + qr^4b \\ +p^4q - 2bpr^3 + 3qr^2bp^2 - 4r^2bq + 8rpb - 2rbp^3 - 2p^3q^2r - 4bp^2q - 2r^3pq^2)x \\ -p^2q^2r^2 + 2pr^3q + 2p^3qr - p^4 - r^4 - 2p^2r^2 - 4qbpr + q^2br^2 + 4bp^2, \\ (-qpr + p^2 + r^2)y + pqx - 2rx - 2p + qr. \end{cases} \quad (TS_8)$$

$$\begin{cases} (-1 + a + b)x^2 + (-qa + q)x - 1 + a - b, \\ (-1 + a + b)y^2 - 1 - a + qxa + b, \\ p, \\ r. \end{cases} \quad (TS_9)$$

$$\begin{cases} (2pr - p^2q)x - r^2 + p^2, \\ py - 1, \\ (-pqr^3 + r^4 + rp^3q - 4r^2 - p^2q^2 + 4rpq - p^4)a + p^2q^2 - 4rpq + 4r^2, \\ (-pqr^3 + r^4 + rp^3q - 4r^2 - p^2q^2 + 4rpq - p^4)b + \\ p^4 + r^4 + 2r^2p^2 + p^2r^2q^2 - 2rp^3q - 2pqr^3. \end{cases} \quad (TS_{10})$$

$$\begin{cases} rx - p, \\ (-p^2r^2 + r^3qp - r^4)y^2 + (p^3r^2 - p^2r^3q + r^4p)y \\ (-pqr^3 + r^4 + rp^3q - 4r^2 - p^2q^2 + 4rpq - p^4)a + p^2q^2 - 4rpq + 4r^2, \\ (-pqr^3 + r^4 + rp^3q - 4r^2 - p^2q^2 + 4rpq - p^4)b + \\ p^4 + r^4 + 2r^2p^2 + p^2r^2q^2 - 2rp^3q - 2pqr^3. \end{cases} \quad (TS_{11})$$

$$(20)$$

Here,

$$\begin{aligned}
a_0 &= -2b + b^2 + a^2 + 1 - br^2a + 2ba - 2a \\
a_1 &= -2bqa - 2a^2q + br^2qa - 2q + 2bq + 4aq + pbr + brpa - b^2rp \\
a_2 &= q^2 + b^2r^2 - bp^2 - qpbr + b^2p^2 - br^2a + 2 - 2b^2 - abrpq + 2a^2 - 4a - 2q^2a + q^2a^2) \\
a_3 &= -b^2rp + brpa - 2a^2q + qp^2b + 2bqa + 4aq + pbr - 2bq - 2q \\
a_4 &= 1 - 2a + 2b + b^2 - bp^2 + a^2 - 2ba, \\
a_5 &= (apr + 2qa - rpb + 2bq - 2q - ar^2q + pr, \\
a_6 &= (-2q^2 + r^2 - 4 + r^2q^2 - pqr)a + br^2 - p^2 - bq^2 + bp^2 + 2q^2 + 4 - pqr - 4b, \\
a_7 &= (6q + pr - 2r^2q)a + pr - 6q - rpb + 2bq + qp^2, \\
a_8 &= 4 - 4a - p^2 + ar^2,
\end{aligned}$$

$$
\begin{aligned}
b_0 &= b(p^2a - p^2 + bp^2 + pqr - qarp + ar^2 - r^2 - br^2)^2, \\
b_1 &= ((1-a-b)x^2 + (qa-q)x + 1 - a + b)((a^2r^3 + 2br^3a - br^5a - 2ar^3 + r^3 + b^2r^3 - 2r^3b)x^3 + \\
&\quad (pr^2 + pa^2r^2 - 2br^3qa + 2r^3bq - 2r^3q - 2par^2 - 2pr^2b + r^4pb + 4ar^3q + bqar^5 - 2r^3a^2q \\
&\quad +2r^2pba + b^2r^2p - r^4pb^2)x^2 + (r^3q^2 + r^5b^2 + rp^2b^2 - 4ar^3 - 2ar^3q^2 + r^3q^2a^2 + \\
&\quad 2a^2r^3 - 2b^2r^3 - 2p^2br + 4par^2q + 2ap^2rb - 2ar^2qbp - 2p^2ar + rp^2 - br^5a + 2pr^2bq + \\
&\quad rp^2a^2 - 2pqr^2 + 2r^3 - 2r^2pa^2q - r^4qbp)x + 4ar^3q + pr^2q^2 + 2p^3ba - 4par^2 + \\
&\quad -2r^3bq - 2p^2qr - 2b^2r^2p + r^4pb + 2pa^2r^2 - 2r^3a^2q - 2p^3a + p^3a^2 + 2pr^2 + p^3 + 2br^3qa \\
&\quad +2qp^2br + 4qarp^2 - 2par^2q^2 - 2p^2a^2rq + pa^2r^2q^2 - 2r^3q - 2p^3b + p^3b^2 - 2p^2brqa), \\
b_2 &= b(-4ar^3 + 4r^3 + ar^5 - 2p^3q + 4rp^2 - 6pqr^2 - 4rp^2b - 4p^2ar + 6par^2q + \\
&\quad 2p^2rq^2 + 2p^2ar^3 + 2p^3bq + 2p^3qa + p^4ar + p^2ar^3q^2 - 2p^2rq^2a - p^2rbq^2 - \\
&\quad 2p^3ar^2q - 2par^4q + 2pr^2bq), \\
b_3 &= ((-1+a+b)x^2 + (-qa+q)x - 1 + a - b)((-par^3 + ar^4q - 2ar^2q - 2r^2bq + 2r^2q - pr^3 + \\
&\quad r^3bp)x^2 + (-r^2p^2a + 2r^3paq + 4ar^2 - r^4q^2a - ar^4 - 2qarp + 2ar^2q^2 - r^4b + r^2bq^2 + r^3pq \\
&\quad +2pqr + 4r^2b - 2qpbr - 2r^2q^2 - 4r^2)x - p^3ar + 2ar^4q - par^3q^2 + 2p^2ar^2q - 2par^3 + 2prq^2a \\
&\quad -2p^2aq - 6ar^2q + 4apr - pr^3 + 4pbr + prbq^2 - 2r^2bq - 2prq^2 + 2qp^2 - 2bqp^2 - 4pr + 6r^2q), \\
b_4 &= r^2p^2(rq^2 + p^2r - 4pq)(p^2 - pqr + r^2 + q^2 - 4), \\
b_5 &= r^2q((rp^2 + rq^2 - r^2pq)x + pr^2 - 4p)((rp^2 + rq^2 - 4pq)x + q^2p - qp^2r + p^3), \\
I_1 &= a_0, I_2 = b_0, I_3 = a_5, I_4 = b_2, I_5 = r, I_6 = rp - 4pq + rq^2, I_7 = p, I_8 = (p^2+q^2)b - p^2. \\
T_1 &= I_0I_1I_2, T_2 = I_0I_2I_3I_4, T_3 = I_0I_2I_4I_5I_6I_7, T_4 = I_0I_2I_4I_6I_7I_8, \\
T_5 &= I_0I_2I_4I_6I_7, T_6 = I_0I_2I_4I_5I_7, T_7 = I_0I_2I_3, T_8 = T_7, T_9 = I_0I_1, T_{10} = I_0I_1, T_{11} = I_0I_1.
\end{aligned}
$$

Appendix B. This appendix gives the explicit formulas for the conditions in the theorems in Section 3.

$$V_{f_1}^{(4,4)}(x > 0) = [a_0 > 0, a_1 < 0, a_2 > 0, a_3 < 0, a_4 > 0] \vee [a_0 < 0, a_1 > 0, a_2 < 0, a_3 > 0, a_4 < 0]$$

$$V_{f_1}^{(4,4)}(g_1 > 0) = [r_{10} > 0, r_{11} < 0, r_{12} > 0, r_{13} < 0, r_{14} > 0] \vee [r_{10} < 0, r_{11} > 0, r_{12} < 0, r_{13} > 0, r_{14} < 0]$$

$$V_{f_1}^{(4,4)}(xg_1 > 0) = [r_{20} > 0, r_{11} < 0, r_{22} > 0, r_{23} < 0, r_{24} > 0] \vee [r_{20} < 0, r_{21} > 0, r_{22} < 0, r_{23} > 0, r_{24} < 0]$$

$$V_{f_1}^{(4,3)}(x > 0) = [a_0 > 0, a_1 0, a_2 < 0, a_3 > 0, a_4 < 0] \vee [a_0 > 0, a_1 < 0, a_2 > 0, a_3 \geq 0, a_4 < 0] \vee [a_0 > 0, a_1 < 0, a_2 \geq 0, a_3 > 0, a_4 < 0] \vee [a_0 > 0, a_1 < 0, a_2 > 0, a_3 < 0, a_4 \leq 0] \vee [a_0 < 0, a_1 \leq 0, a_2 > 0, a_3 < 0, a_4 > 0] \vee [a_0 < 0, a_1 > 0, a_2 \geq 0, a_3 < 0, a_4 > 0] \vee [a_0 < 0, a_1 > 0, a_2 < 0, a_3 \leq 0, a_4 > 0] \vee [a_0 < 0, a_1 > 0, a_2 < 0, a_3 < 0, a_4 \leq 0]$$

$$V_{1}^{(4,3)}(g_1 > 0) = [r_{10} > 0, r_{11} \geq 0, r_{12} < 0, r_{13} > 0, r_{14} < 0] \vee [r_{10} > 0, r_{11} < 0, r_{12} > 0, r_{13} \geq 0, r_{14} < 0] \vee [r_{10} > 0, r_{11} < 0, r_{12} \geq 0, r_{13} > 0, r_{14} < 0] \vee [r_{10} > 0, r_{11} < 0, r_{12} > 0, r_{13} < 0, r_{14} \leq 0] \vee [r_{10} < 0, r_{11} \leq 0, r_{12} > 0, r_{13} < 0, r_{14} > 0] \vee [r_{10} < 0, r_{11} > 0, r_{12} \geq 0, r_{13} < 0, r_{14} > 0] \vee [r_{10} < 0, r_{11} > 0, r_{12} < 0, r_{13} \leq 0, r_{14} > 0] \vee [r_{10} < 0, r_{11} > 0, r_{12} < 0, r_{13} < 0, r_{14} \leq 0]$$

$$V_{1}^{(4,3)}(xg_1 > 0) = [r_{20} > 0, r_{21} \geq 0, r_{22} < 0, r_{23} > 0, r_{24} < 0] \vee [r_{20} > 0, r_{21} < 0, r_{22} > 0, r_{23} \geq 0, r_{24} < 0] \vee [r_{20} > 0, r_{21} < 0, r_{22} \geq 0, r_{23} > 0, r_{24} < 0] \vee [r_{20} > 0, r_{21} < 0, r_{22} > 0, r_{23} < 0, r_{24} \leq 0] \vee [r_{20} < 0, r_{21} \leq 0, r_{22} > 0, r_{23} < 0, r_{24} > 0] \vee [r_{20} < 0, r_{21} > 0, r_{22} \geq 0, r_{23} < 0, r_{24} > 0] \vee [r_{20} < 0, r_{21} > 0, r_{22} < 0, r_{23} \leq 0, r_{24} > 0] \vee [r_{20} < 0, r_{21} > 0, r_{22} < 0, r_{23} < 0, r_{24} \leq 0]$$

$$V_{f_1}^{(4,2)}(x > 0) = [a_0 > 0, a_1 \neq 0, a_2 < 0, a_4 > 0] \vee [a_0 > 0, a_1 < 0, a_2 = 0, a_4 > 0] \vee [a_0 > 0, a_1 < 0, a_3 > 0, a_4 = 0] \vee [a_0 > 0, a_1 < 0, a_2 > 0, a_3 \geq 0, a_4 > 0] \vee [a_0 > 0, a_1 \geq 0, a_2 = 0, a_3 < 0, a_4 > 0] \vee [a_0 > 0, a_1 > 0, a_2 > 0, a_3 < 0, a4 > 0] \vee [a_0 > 0, a_1 = 0, a2 < 0, a_3 < 0, a_4 > 0] \vee [a_0 > 0, a_1 \geq 0, a_2 < 0, a_3 > 0, a_4 = 0] \vee [a_0 > 0, a_1 < 0, a_2 > 0, a_3 = 0, a_4 = 0] \vee [a_0 < 0, a_1 \neq 0, a_2 > 0, a_4 < 0] \vee [a_0 < 0, a_1 > 0, a_2 = 0, a_4 < 0] \vee [a_0 < 0, a_1 > 0, a_3 < 0, a_4 = 0] \vee [a_0 < 0, a_1 > 0, a_2 < 0, a_3 \leq 0, a_4 < 0] \vee [a_0 < 0, a_1 \leq 0, a_2 = 0, a_3 > 0, a_4 < 0] \vee [a_0 < 0, a_1 < 0, a_2 < 0, a_3 > 0, a4 < 0] \vee [a_0 < 0, a_1 = 0, a2 > 0, a_3 > 0, a_4 < 0] \vee [a_0 < 0, a_1 \leq 0, a_2 > 0, a_3 < 0, a_4 = 0] \vee [a_0 < 0, a_1 > 0, a_2 < 0, a_3 = 0, a_4 = 0]$$

$$V_{f_1}^{(4,2)}(g_1 > 0) = [r_{10} > 0, r_{11} \neq 0, r_{12} < 0, r_{14} > 0] \vee [r_{10} > 0, r_{11} < 0, r_{12} = 0, r_{14} > 0] \vee [r_{10} > 0, r_{11} < 0, r_{13} > 0, r_{14} = 0] \vee [r_{10} > 0, r_{11} < 0, r_{12} > 0, r_{13} \geq 0, r_{14} > 0] \vee [r_{10} > 0, r_{11} \geq 0, r_{12} = 0, r_{13} < 0, r_{14} > 0] \vee [r_{10} > 0, r_{11} > 0, r_{12} > 0, r_{13} < 0, r_{14} > 0] \vee [r_{10} > 0, r_{11} = 0, r_{12} < 0, r_{13} < 0, r_{14} > 0] \vee [r_{10} > 0, r_{11} \geq 0, r_{12} < 0, r_{13} > 0, r_{14} = 0] \vee [r_{10} > 0, r_{11} < 0, r_{12} > 0, r_{13} = 0, r_{14} = 0] \vee [r_{10} < 0, r_{11} \neq 0, r_{12} > 0, r_{14} < 0] \vee [r_{10} < 0, r_{11} > 0, r_{12} = 0, r_{14} < 0] \vee [r_{10} < 0, r_{11} > 0, r_{13} < 0, r_{14} = 0] \vee [r_{10} < 0, r_{11} > 0, r_{12} < 0, r_{13} \leq 0, r_{14} < 0] \vee [r_{10} < 0, r_{11} \leq 0, r_{12} = 0, r_{13} > 0, r_{14} < 0] \vee [r_{10} < 0, r_{11} < 0, r_{12} < 0, r_{13} > 0, r_{14} < 0] \vee [r_{10} < 0, r_{11} = 0, r_{12} > 0, r_{13} > 0, r_{14} < 0] \vee [r_{10} < 0, r_{11} \leq 0, r_{12} > 0, r_{13} < 0, r_{14} = 0] \vee [r_{10} < 0, r_{11} > 0, r_{12} < 0, r_{13} = 0, r_{14} = 0]$$

$$V_{f_1}^{(4,2)}(xg_1 > 0) = [r_{20} > 0, r_{21} \neq 0, r_{22} < 0, r_{24} > 0] \vee [r_{20} > 0, r_{21} < 0, r_{22} = 0, r_{24} > 0] \vee [r_{20} > 0, r_{21} < 0, r_{23} > 0, r_{24} = 0] \vee [r_{20} > 0, r_{21} < 0, r_{22} > 0, r_{23} \geq 0, r_{24} > 0] \vee [r_{20} > 0, r_{21} \geq 0, r_{22} = 0, r_{23} < 0, r_{24} > 0] \vee [r_{20} > 0, r_{21} > 0, r_{22} > 0, r_{23} < 0, r_{24} > 0] \vee [r_{20} > 0, r_{21} = 0, r_{22} < 0, r_{23} < 0, r_{24} > 0] \vee [r_{20} > 0, r_{21} \geq 0, r_{22} < 0, r_{23} > 0, r_{24} = 0] \vee [r_{20} > 0, r_{21} < 0, r_{22} > 0, r_{23} = 0, r_{24} = 0] \vee [r_{20} < 0, r_{21} \neq 0, r_{22} > 0, r_{24} < 0] \vee [r_{20} < 0, r_{21} > 0, r_{22} = 0, r_{24} < 0] \vee [r_{20} < 0, r_{21} > 0, r_{23} < 0, r_{24} = 0] \vee [r_{20} < 0, r_{21} > 0, r_{22} < 0, r_{23} \leq 0, r_{24} < 0] \vee [r_{20} < 0, r_{21} \leq 0, r_{22} = 0, r_{23} > 0, r_{24} < 0] \vee [r_{20} < 0, r_{21} < 0, r_{22} < 0, r_{23} > 0, r_{24} < 0] \vee [r_{20} < 0, r_{21} = 0, r_{22} > 0, r_{23} > 0, r_{24} < 0] \vee [r_{20} < 0, r_{21} \leq 0, r_{22} > 0, r_{23} < 0, r_{24} = 0] \vee [r_{20} < 0, r_{21} > 0, r_{22} < 0, r_{23} = 0, r_{24} = 0]$$

$$V_{f_1}^{(4,1)}(x > 0) = [a_0 > 0, a_1 = 0, a_3 \leq 0, a_4 < 0] \vee [a_0 > 0, a_1 > 0, a_2 = 0, a_4 < 0] \vee [a_0 > 0, a_1 > 0, a_2 > 0, a_3 \geq 0, a_4 < 0] \vee [a_0 > 0, a_1 > 0, a_2 > 0, a_3 < 0, a_4 \leq 0] \vee [a_0 > 0, a_1 < 0, a_2 = 0, a_3 \leq 0, a_4 < 0] \vee [a_0 > 0, a_1 < 0, a_2 = 0, a_3 \leq 0, a_4 = 0] \vee [a_0 > 0, a_1 < 0, a_2 < 0, a_3 \leq 0, a_4 = 0] \vee [a_0 > 0, a_1 > 0, a_2 < 0, a_3 \leq 0, a_4 = 0] \vee [a_0 > 0, a_1 < 0, a_2 < 0, a_3 \leq 0, a_4 < 0] \vee [a_0 > 0, a_1 > 0, a_2 < 0, a_3 \leq 0, a_4 < 0] \vee [a_0 > 0, a_1 = 0, a_2 = 0, a_3 < 0, a_4 \leq 0] \vee [a_0 > 0, a_1 = 0, a_2 < 0, a_3 \leq 0, a_4 = 0] \vee [a_0 > 0, a_1 > 0, a_2 = 0, a_3 < 0, a_4 = 0] \vee [a > 0, a_1 = 0, a_2 > 0, a_3 > 0, a_4 < 0] \vee [a_0 > 0, a_1 = 0, a_2 > 0, a_3 < 0, a_4 = 0] \vee [a_0 < 0, a_1 = 0, a_3 \geq 0, a_4 > 0] \vee [a_0 < 0, a_1 < 0, a_2 = 0, a_4 > 0] \vee [a_0 < 0, a_1 < 0, a_2 < 0, a_3 \leq 0, a_4 > 0] \vee [a_0 < 0, a_1 < 0, a_2 < 0, a_3 < 0, a_4 \geq 0] \vee [a_0 < 0, a_1 > 0, a_2 = 0, a_3 \geq 0, a_4 > 0] \vee [a_0 < 0, a_1 > 0, a_2 = 0, a_3 \geq 0, a_4 = 0] \vee [a_0 < 0, a_1 > 0, a_2 > 0, a_3 \geq 0, a_4 = 0] \vee [a_0 < 0, a_1 < 0, a_2 > 0, a_3 \geq 0, a_4 = 0] \vee [a_0 < 0, a_1 < 0, a_2 > 0, a_3 \geq 0, a_4 > 0] \vee [a_0 < 0, a_1 < 0, a_2 > 0, a_3 \geq 0, a_4 > 0] \vee [a_0 < 0, a_1 = 0, a_2 = 0, a_3 > 0, a_4 \geq 0] \vee [a_0 < 0, a_1 = 0, a_2 > 0, a_3 \geq 0, a_4 = 0] \vee [a_0 < 0, a_1 < 0, a_2 = 0, a_3 > 0, a_4 = 0] \vee [a_0 < 0, a_1 = 0, a_2 < 0, a_3 < 0, a_4 > 0] \vee [a_0 < 0, a_1 = 0, a_2 < 0, a_3 > 0, a_4 = 0]$$

$$V_{f_1}^{(4,1)}(g_1 > 0) = [r_{10} > 0, r_{11} = 0, r_{13} \leq 0, r_{14} < 0] \vee [r_{10} > 0, r_{11} > 0, r_{12} = 0, r_{14} < 0] \vee [r_{10} > 0, r_{11} > 0, r_{12} > 0, r_{13} \geq 0, r_{14} < 0] \vee [r_{10} > 0, r_{11} > 0, r_{12} > 0, r_{13} < 0, r_{14} \leq 0] \vee [r_{10} > 0, r_{11} < 0, r_{12} = 0, r_{13} \leq 0, r_{14} < 0] \vee [r_{10} > 0, r_{11} < 0, r_{12} = 0, r_{13} \leq 0, r_{14} = 0] \vee [r_{10} > 0, r_{11} < 0, r_{12} < 0, r_{13} \leq 0, r_{14} = 0] \vee [r_{10} > 0, r_{11} > 0, r_{12} < 0, r_{13} \leq 0, r_{14} = 0] \vee [r_{10} > 0, r_{11} < 0, r_{12} < 0, r_{13} \leq 0, r_{14} < 0] \vee [r_{10} > 0, r_{11} > 0, r_{12} < 0, r_{13} \leq 0, r_{14} < 0] \vee [r_{10} > 0, r_{11} = 0, r_{12} = 0, r_{13} < 0, r_{14} \leq 0] \vee [r_{10} > 0, r_{11} = 0, r_{12} < 0, r_{13} \leq 0, r_{14} = 0] \vee [r_{10} > 0, r_{11} > 0, r_{12} = 0, r_{13} < 0, r_{14} = 0] \vee [r_{10} > 0, r_{11} = 0, r_{12} > 0, r_{13} > 0, r_{14} < 0] \vee [r_{10} > 0, r_{11} = 0, r_{12} > 0, r_{13} < 0, r_{14} = 0] \vee [r_{10} < 0, r_{11} = 0, r_{13} \geq 0, r_{14} > 0] \vee [r_{10} < 0, r_{11} < 0, r_{12} = 0, r_{14} > 0] \vee [r_{10} < 0, r_{11} < 0, r_{12} < 0, r_{13} \leq 0, r_{14} > 0] \vee [r_{10} < 0, r_{11} < 0, r_{12} < 0, r_{13} < 0, r_{14} \geq 0] \vee [r_{10} < 0, r_{11} > 0, r_{12} = 0, r_{13} \geq 0, r_{14} > 0] \vee [r_{10} < 0, r_{11} > 0, r_{12} = 0, r_{13} \geq 0, r_{14} = 0] \vee [r_{10} < 0, r_{11} > 0, r_{12} > 0, r_{13} \geq 0, r_{14} = 0] \vee [r_{10} < 0, r_{11} < 0, r_{12} > 0, r_{13} \geq 0, r_{14} = 0] \vee [r_{10} < 0, r_{11} < 0, r_{12} > 0, r_{13} \geq 0, r_{14} > 0] \vee [r_{10} < 0, r_{11} < 0, r_{12} > 0, r_{13} \geq 0, r_{14} > 0] \vee [r_{10} < 0, r_{11} = 0, r_{12} = 0, r_{13} > 0, r_{14} \geq 0] \vee [r_{10} < 0, r_{11} = 0, r_{12} > 0, r_{13} \geq 0, r_{14} = 0] \vee [r_{10} < 0, r_{11} < 0, r_{12} = 0, r_{13} > 0, r_{14} = 0] \vee [r_{10} < 0, r_{11} = 0, r_{12} < 0, r_{13} < 0, r_{14} > 0] \vee [r_{10} < 0, r_{11} = 0, r_{12} < 0, r_{13} > 0, r_{14} = 0]$$

$V_{f_1}^{(4,1)}(xg_1) > 0) = [r_{20} > 0, r_{21} = 0, r_{23} \leq 0, r_{24} < 0] \vee [r_{20} > 0, r_{21} > 0, r_{22} = 0, r_{24} < 0] \vee [r_{20} > 0, r_{21} > 0, r_{22} > 0, r_{23} \geq 0, r_{24} < 0] \vee [r_{20} > 0, r_{21} > 0, r_{22} > 0, r_{23} < 0, r_{24} \leq 0] \vee [r_{20} > 0, r_{21} < 0, r_{22} = 0, r_{23} \leq 0, r_{24} < 0] \vee [r_{20} > 0, r_{21} < 0, r_{22} = 0, r_{23} \leq 0, r_{24} = 0] \vee [r_{20} > 0, r_{21} < 0, r_{22} < 0, r_{23} \leq 0, r_{24} = 0] \vee [r_{20} > 0, r_{21} > 0, r_{22} < 0, r_{23} \leq 0, r_{24} = 0] \vee [r_{20} > 0, r_{21} < 0, r_{22} < 0, r_{23} \leq 0, r_{24} < 0] \vee [r_{20} > 0, r_{21} > 0, r_{22} < 0, r_{23} \leq 0, r_{24} < 0] \vee [r_{20} > 0, r_{21} = 0, r_{22} = 0, r_{23} < 0, r_{24} \leq 0] \vee [r_{20} > 0, r_{21} = 0, r_{22} < 0, r_{23} \leq 0, r_{24} = 0] \vee [r_{20} > 0, r_{21} > 0, r_{22} = 0, r_{23} < 0, r_{24} = 0] \vee [r_{20} > 0, r_{21} = 0, r_{22} > 0, r_{23} > 0, r_{24} < 0] \vee [r_{20} > 0, r_{21} = 0, r_{22} > 0, r_{23} < 0, r_{24} = 0] \vee [r_{20} < 0, r_{21} = 0, r_{23} \geq 0, r_{24} > 0] \vee [r_{20} < 0, r_{21} < 0, r_{22} = 0, r_{24} > 0] \vee [r_{20} < 0, r_{21} < 0, r_{22} < 0, r_{23} \leq 0, r_{24} > 0] \vee [r_{20} < 0, r_{21} < 0, r_{22} < 0, r_{23} < 0, r_{24} \geq 0] \vee [r_{20} < 0, r_{21} > 0, r_{22} = 0, r_{23} \geq 0, r_{24} > 0] \vee [r_{20} < 0, r_{21} > 0, r_{22} = 0, r_{23} \geq 0, r_{24} = 0] \vee [r_{20} < 0, r_{21} > 0, r_{22} > 0, r_{23} \geq 0, r_{24} = 0] \vee [r_{20} < 0, r_{21} < 0, r_{22} > 0, r_{23} \geq 0, r_{24} = 0] \vee [r_{20} < 0, r_{21} < 0, r_{22} > 0, r_{23} \geq 0, r_{24} > 0] \vee [r_{20} < 0, r_{21} < 0, r_{22} > 0, r_{23} \geq 0, r_{24} > 0] \vee [r_{20} < 0, r_{21} = 0, r_{22} = 0, r_{23} > 0, r_{24} \geq 0] \vee [r_{20} < 0, r_{21} = 0, r_{22} > 0, r_{23} \geq 0, r_{24} = 0] \vee [r_{20} < 0, r_{21} < 0, r_{22} = 0, r_{23} > 0, r_{24} = 0] \vee [r_{20} < 0, r_{21} = 0, r_{22} < 0, r_{23} < 0, r_{24} > 0] \vee [r_{20} < 0, r_{21} = 0, r_{22} < 0, r_{23} > 0, r_{24} = 0]$

$V_{f_1}^{(2,2)} = [D_2 > 0, D_3 \geq 0, D_4 < 0] \vee [D_2 \geq 0, D_3 < 0, D_4 < 0] \vee [D_2 < 0, D_3 < 0, D_4 < 0]$

$V_{f_1}^{(2,2)}(x > 0) = [A_1 > 0, A_2 > 0, A_3 > 0, A_4 < 0] \vee [A_1 > 0, A_3 < 0, A_4 < 0] \vee [A_1 = 0, A_2 < 0, A_3 < 0, A_4 < 0]$

$V_{f_1}^{(2,2)}(g_1 > 0) = [B_1 > 0, B_2 > 0, B_3 > 0, B_4 < 0] \vee [B_1 > 0, B_3 < 0, B_4 < 0] \vee [B_1 = 0, B_2 < 0, B_3 < 0, B_4 < 0]$

$V_{f_1}^{(2,2)}(xg_1 > 0) = [C_1 > 0, C_2 > 0, C_3 > 0, C_4 < 0] \vee [C_1 > 0, C_3 < 0, C_4 < 0] \vee [C_1 = 0, C_2 < 0, C_3 < 0, C_4 < 0]$

$V_{f_1}^{(2,1)}(x > 0) = [A_1 = 0, A_2 > 0, A_3 \leq 0, A_4 < 0] \vee [A_1 = 0, A_2 \leq 0, A_3 = 0, A_4 < 0] \vee [A_1 > 0, A_2 = 0, A_3 > 0, A_4 < 0] \vee [A_1 = 0, A_2 > 0, A_3 > 0, A_4 < 0] \vee [A_1 > 0, A_2 < 0, A_3 = 0, A_4 < 0] \vee [A_1 > 0, A_2 = 0, A_3 < 0, A_4 < 0] \vee [A_1 < 0, A_2 < 0, A_3 = 0, A_4 < 0] \vee [A_1 < 0, A_2 = 0, A_3 < 0, A_4 < 0]$

$V_{f_1}^{(2,1)}(g_1 > 0) = [B_1 = 0, B_2 > 0, B_3 \leq 0, B_4 < 0] \vee [B_1 = 0, B_2 \leq 0, B_3 = 0, B_4 < 0] \vee [B_1 > 0, B_2 = 0, B_3 > 0, B_4 < 0] \vee [B_1 = 0, B_2 > 0, B_3 > 0, B_4 < 0] \vee [B_1 > 0, B_2 < 0, B_3 = 0, B_4 < 0] \vee [B_1 > 0, B_2 = 0, B_3 < 0, B_4 < 0] \vee [B_1 < 0, B_2 < 0, B_3 = 0, B_4 < 0] \vee [B_1 < 0, B_2 = 0, B_3 < 0, B_4 < 0]$

$V_{f_1}^{(2,1)}(xg_1 > 0) = [C_1 = 0, C_2 > 0, C_3 \leq 0, C_4 < 0] \vee [C_1 = 0, C_2 \leq 0, C_3 = 0, C_4 < 0] \vee [C_1 > 0, C_2 = 0, C_3 > 0, C_4 < 0] \vee [C_1 = 0, C_2 > 0, C_3 > 0, C_4 < 0] \vee [C_1 > 0, C_2 < 0, C_3 = 0, C_4 < 0] \vee [C_1 > 0, C_2 = 0, C_3 < 0, C_4 < 0] \vee [C_1 < 0, C_2 < 0, C_3 = 0, C_4 < 0] \vee [C_1 < 0, C_2 = 0, C_3 < 0, C_4 < 0]$

$V_{f_2}^{(3,3)}(x > 0) = [a_1 > 0, a_2 < 0, a_3 > 0, a_4 < 0] \vee [a_1 < 0, a_2 > 0, a_3 < 0, a_4 > 0]$

$V_{f_2}^{(3,3)}(g_1 > 0) = [r_{71} > 0, r_{72} < 0, r_{73} > 0, r_{74} < 0] \vee [r_{71} < 0, r_{72} > 0, r_{73} < 0, r_{74} > 0]$

$V_{f_2}^{(3,3)}(xg_1 > 0) = [r_{81} > 0, r_{82} < 0, r_{83} > 0, r_{84} < 0] \vee [r_{81} < 0, r_{82} > 0, r_{83} < 0, r_{84} > 0]$

$V_{f_2}^{(3,2)}(x > 0) = [a_1 > 0, a_2 < 0, a_3 \geq 0, a_4 > 0] \vee [a_1 > 0, a_2 \geq 0, a_3 < 0, a_4 > 0] \vee [a_1 > 0, a_2 < 0, a_3 > 0, a_4 = 0] \vee [a_1 < 0, a_2 > 0, a_3 \leq 0, a_4 < 0] \vee [a_1 < 0, a_2 \leq 0, a_3 > 0, a_4 < 0] \vee [a_1 < 0, a_2 > 0, a_3 < 0, a_4 = 0]$

$V_{f_2}^{(3,2)}(g_1 > 0) = [r_{71} > 0, r_{72} < 0, r_{73} \geq 0, r_{74} > 0] \vee [r_{71} > 0, r_{72} \geq 0, r_{73} < 0, r_{74} > 0] \vee [r_{71} > 0, r_{72} < 0, r_{73} > 0, r_{74} = 0] \vee [r_{71} < 0, r_{72} > 0, r_{73} \leq 0, r_{74} < 0] \vee [r_{71} < 0, r_2 \leq 0, r_{73} > 0, r_{74} < 0] \vee [r_{71} < 0, r_{72} > 0, r_{73} < 0, r_{74} = 0]$

$V_{f_2}^{(3,2)}(xg_1 > 0) = [r_{81} > 0, r_{82} < 0, r_{83} \geq 0, r_{84} > 0] \vee [r_{81} > 0, r_{82} \geq 0, r_{83} < 0, r_{84} > 0] \vee [r_{81} > 0, r_{82} < 0, r_{83} > 0, r_{84} = 0] \vee [r_{81} < 0, r_{82} > 0, r_{83} \leq 0, r_{84} < 0] \vee [r_{81} < 0, r_{82} \leq 0, r_{83} > 0, r_{84} < 0] \vee [r_{81} < 0, r_{82} > 0, r_{83} < 0, r_{84} = 0]$

$V_{f_2}^{(3,1)}(x > 0) = [a_1 > 0, a_3 < 0, a_4 = 0] \vee [a_1 > 0, a_2 \leq 0, a_3 < 0, a_4 < 0] \vee [a_1 > 0, a_2 \leq 0, a_3 = 0, a_4 < 0] \vee [a_1 > 0, a_2 > 0, a_3 \leq 0, a_4 < 0] \vee [a_1 > 0, a_2 < 0, a_3 = 0, a_4 = 0] \vee [a_1 > 0, a_2 > 0, a_3 < 0, a_4 = 0] \vee [a_1 < 0, a_3 > 0, a_4 = 0] \vee [a_1 < 0, a_2 \geq 0, a_3 > 0, a_4 > 0] \vee [a_1 < 0, a_2 \geq 0, a_3 = 0, a_4 > 0] \vee [a_1 < 0, a_2 < 0, a_3 \geq 0, a_4 > 0] \vee [a_1 < 0, a_2 > 0, a_3 = 0, a_4 = 0] \vee [a_1 < 0, a_2 < 0, a_3 > 0, a_4 = 0]$

$V_{f_2}^{(3,1)}(g_1 > 0) = [r_{71} > 0, r_{73} < 0, r_{74} = 0] \vee [r_{71} > 0, r_{72} \leq 0, r_{73} < 0, r_{74} < 0] \vee [r_{71} > 0, r_{72} \leq 0, r_{73} = 0, r_{74} < 0] \vee [r_{71} > 0, r_{72} > 0, r_{73} \leq 0, r_{74} < 0] \vee [r_{71} > 0, r_{72} < 0, r_{73} = 0, r_{74} = 0] \vee [r_{71} > 0, r_{72} > 0, r_{73} < 0, r_{74} = 0] \vee [r_{71} < 0, r_{73} > 0, r_{74} = 0] \vee [r_{71} < 0, r_{72} \geq 0, r_{73} > 0, r_{74} > 0] \vee [r_{71} < 0, r_{72} \geq 0, r_{73} = 0, r_{74} > 0] \vee [r_{71} < 0, r_{72} < 0, r_{73} \geq 0, r_{74} > 0] \vee [r_{71} < 0, r_{72} > 0, r_{73} = 0, r_{74} = 0] \vee [r_{71} < 0, r_{72} < 0, r_{73} > 0, r_{74} = 0]$

$V_{f_2}^{(3,1)}(xg_1 > 0) = [r_{81} > 0, r_{83} < 0, r_{84} = 0] \vee [r_{81} > 0, r_{82} \leq 0, r_{83} < 0, r_{84} < 0] \vee [r_{81} > 0, r_{82} \leq 0, r_{83} = 0, r_{84} < 0] \vee [r_{81} > 0, r_{82} > 0, r_{83} \leq 0, r_{84} < 0] \vee [r_{81} > 0, r_{82} < 0, r_{83} = 0, r_{84} = 0] \vee [r_{81} > 0, r_{82} > 0, r_{83} < 0, r_{84} = 0] \vee [r_{81} < 0, r_{83} > 0, r_{84} = 0] \vee [r_{81} < 0, r_{82} \geq 0, r_{83} > 0, r_{84} > 0] \vee [r_{81} < 0, r_{82} \geq 0, r_{83} = 0, r_{84} > 0] \vee [r_{81} < 0, r_{82} < 0, r_{83} \geq 0, r_{84} > 0] \vee [r_{81} < 0, r_{82} > 0, r_{83} = 0, r_{84} = 0] \vee [r_{81} < 0, r_{82} < 0, r_{83} > 0, r_{84} = 0]$

$V_{f_2}^{(1,1)}(x > 0) = [a_1 > 0, a_3 < 0, a_4 = 0] \vee [a_1 > 0, a_2 \leq 0, a_3 < 0, a_4 < 0] \vee [a_1 > 0, a_2 \leq 0, a_3 = 0, a_4 < 0] \vee [a_1 > 0, a_2 > 0, a_3 \leq 0, a_4 < 0] \vee [a_1 > 0, a_2 < 0, a_3 = 0, a_4 = 0] \vee [a_1 > 0, a_2 > 0, a_3 < 0, a_4 = 0] \vee [a_1 < 0, a_3 > 0, a_4 = 0] \vee [a_1 < 0, a_2 \geq 0, a_3 > 0, a_4 > 0] \vee [a_1 < 0, a_2 \geq 0, a_3 = 0, a_4 > 0] \vee [a_1 < 0, a_2 < 0, a_3 \geq 0, a_4 > 0] \vee [a_1 < 0, a_2 > 0, a_3 = 0, a_7 = 0] \vee [a_1 < 0, a_2 < 0, a_3 > 0, a_4 = 0]$

$V_{f_2}^{(1,1)}(g_1 > 0) = [r_{71} > 0, r_{73} < 0, r_{74} = 0] \vee [r_{71} > 0, r_{72} \leq 0, r_{73} < 0, r_{74} < 0] \vee [r_{71} > 0, r_{72} \leq 0, r_{73} = 0, r_{74} < 0] \vee [r_{71} > 0, r_{72} > 0, r_{73} \leq 0, r_{74} < 0] \vee [r_{71} > 0, r_{72} < 0, r_{73} = 0, r_{74} = 0] \vee [r_{71} > 0, r_{72} > 0, r_{73<0}, r_{74} = 0] \vee [r_{71} < 0, r_{73} > 0, r_{74} = 0] \vee [r_{71} < 0, r_{72} \geq$

$0, r_{73} > 0, r_{74} > 0] \vee [r_{71} < 0, r_{72} \geq 0, r_{73} = 0, r_{74} > 0] \vee [r_{71} < 0, r_{72} < 0, r_{73} \geq 0, r_{74} > 0] \vee [r_{71} < 0, r_{72} > 0, r_{73} = 0, r_{74} = 0] \vee [r_{71} < 0, r_{72} < 0, r_{73} > 0, r_{74} = 0]$

$v_{f_2}^{(1,1)}(xg_1 > 0) = [r_{81} > 0, r_{83} < 0, r_{84} = 0] \vee [r_{81} > 0, r_{82} \leq 0, r_{83} < 0, r_{84} < 0] \vee [r_{81} > 0, r_{82} \leq 0, r_{83} = 0, r_{84} < 0] \vee [r_{81} > 0, r_{82} > 0, r_{83} \leq 0, r_{84} < 0] \vee [r_{81} > 0, r_{82} < 0, r_{83} = 0, r_{84} = 0] \vee [r_{81} > 0, r_{82} > 0, r_{83<0}, r_{84} = 0] \vee [r_{81} < 0, r_{83} > 0, r_{84} = 0] \vee [r_{81} < 0, r_{82} \geq 0, r_{83} > 0, r_{84} > 0] \vee [r_{81} < 0, r_{82} \geq 0, r_{83} = 0, r_{84} > 0] \vee [r_{81} < 0, r_{82} < 0, r_{83} \geq 0, r_{84} > 0] \vee [r_{81} < 0, r_{82} > 0, r_{83} = 0, r_{84} = 0] \vee [r_{81} < 0, r_{82} < 0, r_{83} > 0, r_{84} = 0]$

$V_{f_3}^{(2,2)}(x > 0) = [a_2 > 0, a_3 < 0, a_4 > 0] \vee [a_2 < 0, a_3 > 0, a_4 < 0]$

$V_{f_3}^{(2,2)}(g_1 > 0) = [r_{92} > 0, r_{93} < 0, r_{94} > 0] \vee [r_{92} < 0, r_{93} > 0, r_{94} < 0]$

$V_{f_3}^{(2,2)}(xg_1 > 0) = [r_{102} > 0, r_{103} < 0, r_{104} > 0] \vee [r_{102} < 0, r_{103} > 0, r_{104} < 0]$

$V_{f_3}^{(2,1)}(x > 0) = [a_2 > 0, a_3 < 0, a_4 \leq 0] \vee [a_2 > 0, a_3 \geq 0, a_4 < 0] \vee [a_2 < 0, a_3 > 0, a_4 \geq 0] \vee [a_2 < 0, a_3 \geq > 0, a_4 > 0]$

$V_{f_3}^{(2,1)}(g_1 > 0) = [r_{92} > 0, r_{93} < 0, r_{94} \leq 0] \vee [r_{92} > 0, r_{93} \geq 0, r_{94} < 0] \vee [r_{92} < 0, r_{93} > 0, r_{94} \geq 0] \vee [r_{92} < 0, r_{93} \geq 0, r_{94} 0]$

$V_{f_3}^{(2,1)}(xg_1 > 0) = [r_{102} > 0, r_{103} < 0, r_{104} \leq 0] \vee [r_{102} > 0, r_{103} \geq 0, r_{104} < 0] \vee [r_{102} < 0, r_{103} > 0, r_{104} \geq 0] \vee [r_{102} < 0, r_{103} \geq 0, r_{104} > 0]$

$V_{Q_1}^{(2,2)}(x > 0) = [q_{10} > 0, q_{11} < 0, q_{12} > 0] \vee [q_{10} < 0, q_{11} > 0, q_{12} < 0]$

$V_{Q_1}^{(2,2)}(g_1 > 0) = [r_{30} > 0, r_{31} < 0, r_{32} > 0] \vee [r_{30} < 0, r_{31} > 0, r_{32} < 0]$

$V_{Q_1}^{(2,2)}(xg_1 > 0) = [r_{40} > 0, r_{41} < 0, r_{42} > 0] \vee [r_{40} < 0, r_{41} > 0, r_{42} < 0]$

$V_{Q_1}^{(2,1)}(x > 0) = [q_{10} > 0, q_{12} < 0] \vee [q_{10} < 0, q_{12} > 0] \vee [q_{10} > 0, q_{11} < 0, q_{12} = 0] \vee [q_{10} < 0, q_{11} > 0, q_{12} = 0]$

$V_{Q_1}^{(2,1)}(g_1 > 0) = [r_{30} > 0, r_{32} < 0] \vee [r_{30} < 0, r_{32} > 0] \vee [r_{30} > 0, r_{31} < 0, r_{32} = 0] \vee [r_{30} < 0, r_{31} > 0, r_{32} = 0]$

$V_{Q_1}^{(2,1)}(xg_1 > 0) = [r_{40} > 0, r_{42} < 0] \vee [r_{40} < 0, r_{42} > 0] \vee [r_{40} > 0, r_{41} < 0, r_{42} = 0] \vee [r_{40} < 0, r_{41} > 0, r_{42} = 0]$

$V_{H_2}^{(2,2)}(x > 0) = [h_{10} > 0, h_{11} < 0, h_{12} > 0] \vee [h_{10} < 0, h_{11} > 0, h_{12} < 0]$

$V_{H_2}^{(2,2)}(g_1 > 0) = [r_{50} > 0, r_{51} < 0, r_{52} > 0] \vee [r_{50} < 0, r_{51} > 0, r_{52} < 0]$

$V_{H_2}^{(2,2)}(xg_1 > 0) = [r_{60} > 0, r_{61} < 0, r_{62} > 0] \vee [r_{60} < 0, r_{61} > 0, r_{62} < 0]$

$V_{H_2}^{(2,1)}(x > 0) = [h_{20} > 0, h_{22} < 0] \vee [h_{20} < 0, h_{22} > 0] \vee [h_{20} > 0, h_{21} < 0, h_{22} = 0] \vee [h_{20} < 0, h_{21} > 0, h_{22} = 0]$

$V_{H_2}^{(2,1)}(g_1 > 0) = [r_{50} > 0, r_{52} < 0] \vee [r_{50} < 0, r_{52} > 0] \vee [r_{50} > 0, r_{51} < 0, r_{52} = 0] \vee [r_{50} < 0, r_{51} > 0, r_{52} = 0]$

$V_{H_2}^{(2,1)}(xg_1 > 0) = [r_{60} > 0, r_{62} < 0] \vee [r_{60} < 0, r_{62} > 0] \vee [r_{60} > 0, r_{61} < 0, r_{62} = 0] \vee [r_{60} < 0, r_{61} > 0, r_{62} = 0]$