

# A Collection of 130 Geometric Constraint Problems and Their Solutions

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This report is a collection of 134 geometric constraint problems solved by our constrain solver. They are collected by searching more than a thousand of geometric constraint problems, so that each of them has different kind of loops. The method of solving these constraint problems can be found in

X.S. Gao and S.C. Chou, Solving Geometric Constraint Systems, I. A Global Propagation Approach, *Computer-Aided Design*, Vol. 30, No. 1, 47-54, 1998.

This collection is organized as follows. Section 1 introduces the terminologies. Section 2 is the collection of the 134 constraint problems. Section 3 reports the timing statistics.

## 1 Terminology

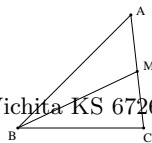
In this section, we will explain the symbols used in this collection. For each constraint problem, we will give its diagram, its input to the constraint solver, and the output of the constraint solver. The input for a constraint problem is the constraint descriptions of a diagram or mechanic design. The output is a construction sequence which shows how to construction the diagram using ruler and compass.

Consider the following example.

**Example 1** Draw a triangle  $ABC$  if we know  $|BC|$ ,  $\angle(BAC)$ , and the length of the median  $BM$ .

We can describe the above constraint problem as follows.

The constraints:  
[dis, $B,C,d1$ ],



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[angle,C,A,B,a],  
 [mid,M,A,C],  
 [dis,B,M,d2].

The constraints used by us are in the following table.

constraints	geometric meaning
[dis,A,B,d]	$ AB  = d$
[angle,A,B,C,a]	$\angle(ABC) = a$
[pl_dis,P,A,B,d]	distance from P to line $AB$ is $d$
[cong,A,B,C,D]	$ AB  =  CD $
[eqangle,A,B,C,P,Q,R]	$\angle(ABC) = \angle(PQR)$
[coll,A,B,C]	$A, B, C$ are collinear
[para,A,B,P,Q]	$AB$ is parallel to $PQ$
[perp,A,B,P,Q]	$AB$ is perpendicular to $PQ$
[mid,M,A,B]	$M$ is the midpoint of $AB$
[sratio,A,B,P,Q,r <sub>1</sub> ,r <sub>2</sub> ]	$ AB / PQ  = r_1/r_2$
[sq_sum,B,A,C,d]	$ AB ^2 +  AC ^2 = d$
[sq_dif,B,A,C,d]	$ AB ^2 -  AC ^2 = d$
[aratio,A,B,C,P,Q,R,r <sub>1</sub> ,r <sub>2</sub> ]	$\text{area}(ABC)/\text{area}(PQR) = r_1/r_2$

The output of the constraint solver is a *construction sequence*  $C_1, C_2, \dots, C_p$  such that each construction  $C_i$  introduces a point. We use the following constructions.

constraints	geometric meaning
[point,P <sub>1</sub> , ..., P <sub>k</sub> ]	Take $k$ free points
[on,P,ob <sub>1</sub> ]	Take a free point on the geometric object $ob_1$
[inter,P,ob <sub>1</sub> ,ob <sub>2</sub> ]	Take the intersection of $ob_1$ and $ob_2$
[mid,P,A,B]	Take the midpoint $P$ of $AB$
[sym,P,A,B]	Take a point $P$ s.t. $A$ is the midpoint of $PB$
[foot,P,A,B,C]	Take the foot drawn from $A$ to $BC$
[mratio,P,A,B,r <sub>1</sub> ,r <sub>2</sub> ]	Take a point on $AB$ such that $\frac{AP}{PB} = \frac{r_1}{r_2}$
[lratio,P,A,B,r <sub>1</sub> ,r <sub>2</sub> ]	Take a point on $AB$ such that $\frac{AP}{AB} = \frac{r_1}{r_2}$

The geometric object is either a line or a circle. Lines and circles are represented as functions of their characteristic points. The following are the lines used in the method:

1. (line,  $P, Q$ ) is the line passing through points  $P$  and  $Q$ .
2. (pline,  $R, P, Q$ ) is the line passing through  $R$  and parallel to line  $PQ$ .
3. (tline,  $R, P, Q$ ) is the line passing through  $R$  and perpendicular to line  $PQ$ .
4. (bline,  $P, Q$ ) is the perpendicular bisector of segment  $PQ$ .
5. (d\_line,  $P, Q, d$ ) is the locus of the points having distance  $d$  to  $PQ$ , i.e. it represents two lines.
6. (m\_line,  $P, Q, R$ ) is the line passing through the midpoints of  $PQ$  and  $PR$ .

7. (bi\_line,  $P, Q, R$ ) is the bisector of angle  $\angle(PQR)$ .
8. (nline,  $P, Q, \alpha$ ) is the line passing through  $P$  and forming with  $PQ$  an angle  $\alpha$ .
9. (aline,  $P, Q, A, B, C$ ) is the same as (nline,  $P, Q, \angle(ABC)$ ).
10. (ta\_line,  $P, O, A$ ) is the one of the tangent lines of circle (pcir,  $O, A$ ) drawn from point  $P$ .
11. (taa\_line,  $P, O_1, A_1, O_2, A_2$ ) is one of the co-tangent lines of circles (pcir,  $O_1, A_1$ ) and (pcir,  $O_2, A_2$ ).

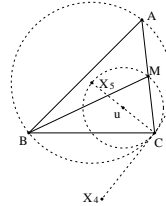
It is clear that all types of lines can be transformed to the form (LINE,  $P, Q$ ). The reason we use complicated line forms is to simplify the output, i.e., the construction sequence of the configuration. If using one type of line only then we need to introduce many auxiliary points. This may complicate the geometric meaning of the configuration and lead to obscured constructions.

We also use the following forms of circles.

1. (pcir,  $O, P$ ) is the circle with center  $O$  and passing through  $P$ .
2. (rcir,  $O, r$ ) is the circle with center  $O$  and radius  $r$ .
3. (a\_cir,  $P, Q, \alpha$ ) is the circle (or an arc) consisting of the points  $R$  satisfying  $\angle PRQ = \alpha$ .
4. (c\_cir,  $P, Q, R$ ) is the circle passing through three points  $P, Q$ , and  $R$ .
5. (di\_cir,  $P, Q$ ) is the circle with  $PQ$  as a diameter.
6. (ac\_cir,  $O, P, a$ ) is the locus of the points which form a constant angles  $a$  with circle (pcir,  $O, A$ ).

The construction sequence for Example 1 found by our constraint solver is given below.

```
[point, B]
[on, C, [rcir, B, d1]]
[on, X4, [nline, C, B, a1]]
[inter, X5, [bline, C, B], [tline, C, C, X4]]
[inter, M, [di_cir, X5, C], [rcir, B, d2]]
[sym, A, M, C]
```



There are two restrictions for the construction [inter,  $P, o_1, o_2$ ] when at least one of  $o_1$  and  $o_2$  is a circle. In this case, there will be two intersection points. We make the following assumptions on this construction.

- If  $o_1$  and  $o_2$  have a common point  $Q$  which has already been constructed before this construction then point  $P$  will be the intersection point of  $o_1$  and  $o_2$  which is different from  $Q$ . The following constructions use this rule.

[inter,  $P$ , [line,  $A, B$ ], [pcir,  $O, A$ ]]. Point  $P$  is different from  $A$ .  
 [inter,  $P$ , [pcir,  $U, A$ ], [pcir,  $O, A$ ]] Point  $P$  is different from  $A$ .  
 [inter,  $P$ , [nline,  $A, B, a$ ], [a\_cir,  $A, C, b$ ]] Point  $P$  is different from  $A$ .

- We also use constructions of the following form

[inter,  $P$ ,  $o_1$ ,  $o_2$ ,  $cs$ ]

where  $cs$  are constraints. The above construction means  $P$  is one of the intersection points of  $o_1$  and  $o_2$  which satisfies the constraints  $cs$ .

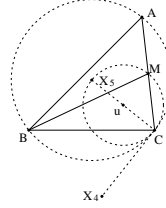
## 2 A Collection of Geometric Constraint Problems

**Example 2** Draw a triangle  $ABC$  if we know  $|BC|$ ,  $\angle(BAC)$ , and the length of the median  $BM$ .

Formally, the constraints are: [dis,  $B, C$ ,  $d1$ ], [angle,  $C, A, B$ ,  $a1$ ], [mid,  $M, A, C$ ], [dis,  $B, M$ ,  $d2$ ].

The construction sequence is

[point,  $B$ ]  
 [on,  $C$ , [rcir,  $B, d1$ ]]  
 [on,  $X_4$ , [nline,  $C, B, a1$ ]]  
 [inter,  $X_5$ , [bline,  $C, B$ ], [tline,  $C, C, X_4$ ]]  
 [inter,  $M$ , [di\_cir,  $X_5, C$ ], [rcir,  $B, d2$ ]]  
 [sym,  $A, M, C$ ]

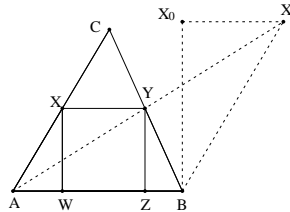


**Example 3** For a given triangle  $ABC$ , draw a square such that two neighboring vertices of the square are on side  $AB$  and the other two vertices are on  $AC$  and  $BC$  respectively.

The constraints: [coll,  $Y, B, C$ ], [coll,  $X, A, C$ ], [coll,  $W, A, B$ ], [coll,  $Z, A, B$ ], [cong,  $Y, X, Y, Z$ ], [perp,  $X, Y, Y, Z$ ], [para,  $W, X, Y, Z$ ], [para,  $X, Y, A, B$ ]. The following points and their construction order are given:  $A, B, C$ .

The construction sequence is

[point,  $A, B, C$ ]  
 [inter,  $X_0$ , [tline,  $B, A, B$ ], [pcir,  $B, A$ ]]  
 [inter,  $X_1$ , [pline,  $B, A, C$ ], [pline,  $X_0, A, B$ ]]  
 [inter,  $Y$ , [line,  $A, X_1$ ], [line,  $B, C$ ]]  
 [inter,  $X$ , [line,  $A, C$ ], [pline,  $Y, A, B$ ]]  
 [inter,  $W$ , [line,  $A, B$ ], [tline,  $X, X, Y$ ]]  
 [inter,  $Z$ , [line,  $A, B$ ], [tline,  $Y, X, Y$ ]]

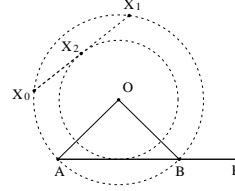


**Example 4** In a given circle, draw a chord which has a fixed length and passes through a known point.

The constraints:  $[dis, O, A, a1]$ ,  $[dis, O, B, a1]$ ,  $[dis, A, B, a2]$ ,  $[coll, P, A, B]$ . The following points and their construction order are given:  $O, P$ .

The construction sequence is

$[point, O, P]$   
 $[on, X_0, [rcir, O, a1]]$   
 $[inter, X_1, [pcir, O, X_0], [rcir, X_0, a2]]$   
 $[mid, X_2, X_0, X_1]$   
 $[inter, A, [ta\_line, P, O, X_2], [rcir, O, a1]]$   
 $[inter, B, [line, P, A], [rcir, O, a1]]$

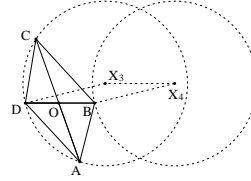


**Example 5** Draw a quadrilateral  $ABCD$  if we know  $AC, BD, \angle(ABC), \angle(ADC)$ , and the angle between its two diagonals  $AC$  and  $BD$ .

The constraints:  $[dis, A, C, d1]$ ,  $[dis, B, D, d2]$ ,  $[angle, A, B, C, a1]$ ,  $[angle, A, D, C, a2]$ ,  $[coll, O, A, C]$ ,  $[coll, O, B, D]$ ,  $[angle, A, O, B, a3]$ .

The construction sequence is

$[point, A, X_0]$   
 $[inter, C, [line, A, X_0], [rcir, A, d1]]$   
 $[on, X_1, [nline, A, X_0, a3]]$   
 $[on, X_2, [nline, A, C, a2]]$   
 $[inter, X_3, [bline, A, C], [tline, A, A, X_2]]$   
 $[inter, X_4, [pline, X_3, A, X_1], [rcir, X_3, d2]]$   
 $[inter, B, [a\_cir, A, C, a1], [dcir, X_4, X_3, A]]$   
 $[inter, O, [line, A, C], [pline, B, X_1, A]]$   
 $[inter, D, [line, O, B], [pline, X_3, B, X_4]]$

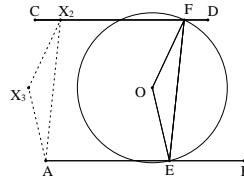


**Example 6** Draw a circle with a known center and cutting a segment with known length between a fixed pair of parallel lines.

The constraints:  $[para, A, B, C, D]$ ,  $[coll, E, A, B]$ ,  $[coll, F, C, D]$ ,  $[dis, E, F, d1]$ ,  $[cong, O, E, O, F]$ . The following points and their construction order are given:  $O, A, B, C, D$ .

The construction sequence is

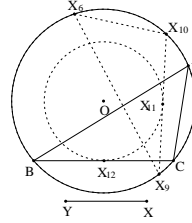
$[point, O, A, B, C]$   
 $[on, D, [pline, C, A, B]]$   
 $[inter, X_2, [line, C, D], [rcir, A, d1]]$   
 $[inter, X_3, [bline, A, X_2], [pline, O, A, B]]$   
 $[inter, E, [line, A, B], [pline, O, A, X_3]]$   
 $[inter, F, [line, C, D], [pline, O, X_3, X_2]]$



**Example 7** Draw a circumscribed triangle  $ABC$  in a given circle  $O$ , if  $|AB|, |AC|$  are known, and  $BC$  is parallel to a given line  $XY$ . The constraints:  $[\text{dis}, O, A, \text{d1}]$ ,  $[\text{dis}, O, B, \text{d1}]$ ,  $[\text{dis}, O, C, \text{d1}]$ ,  $[\text{dis}, A, B, \text{d2}]$ ,  $[\text{dis}, A, C, \text{d3}]$ ,  $[\text{para}, B, C, X, Y]$ . The following points and their construction order are given:  $O, X, Y$ .

The construction sequence is

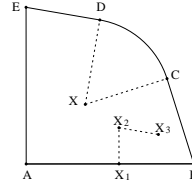
```
[point, O, X, Y]
[on, X6, [rcir, O, d1]]
[inter, X9, [pcir, O, X6], [rcir, X6, d2]]
[inter, X10, [pcir, O, X6], [rcir, X6, d3]]
[mid, X11, X9, X10]
[inter, X12, [tline, O, X, Y], [pcir, O, X11]]
[inter, B, [tline, X12, X12, O], [rcir, O, d1]]
[inter, C, [pline, B, X, Y], [rcir, O, d1]]
[inter, A, [rcir, C, d3], [rcir, O, d1], [dis, A, B, d2]]
```



**Example 8** The constraints:  $[\text{dis}, B, C, \text{d1}]$ ,  $[\text{dis}, X, D, \text{d3}]$ ,  $[\text{dis}, E, D, \text{d2}]$ ,  $[\text{dis}, X, C, \text{d3}]$ ,  $[\text{perp}, X, C, C, B]$ ,  $[\text{perp}, X, D, D, E]$ ,  $[\text{angle}, A, B, C, \text{a1}]$ ,  $[\text{angle}, D, E, A, \text{a2}]$ ,  $[\text{perp}, B, A, A, E]$ .

The construction sequence is

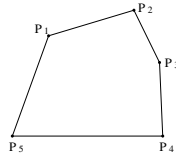
```
[point, B, X0]
[inter, C, [line, B, X0], [rcir, B, d1]]
[inter, X, [tline, C, C, B], [rcir, C, d3]]
[on, X1, [nline, B, C, a1]]
[on, X2, [tline, X1, X1, B]]
[on, X3, [nline, X2, X1, a2]]
[inter, D, [tline, X, X3, X2], [rcir, X, d3]]
[inter, E, [tline, D, X, D], [rcir, D, d2]]
[inter, A, [nline, B, C, a1], [nline, E, D, a2]]
```



**Example 9** The constraints:  $[\text{dis}, P4, P5, \text{d1}]$ ,  $[\text{dis}, P1, P5, \text{d1}]$ ,  $[\text{dis}, P1, P2, \text{d2}]$ ,  $[\text{dis}, P2, P3, \text{d3}]$ ,  $[\text{angle}, P1, P5, P4, \text{a1}]$ ,  $[\text{angle}, P2, P3, P4, \text{a1}]$ ,  $[\text{angle}, P3, P4, P5, \text{a1}]$ .

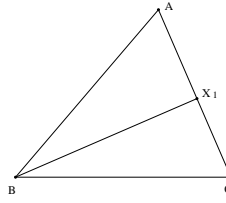
The construction sequence is

```
[point, P4, X0]
[inter, P5, [line, P4, X0], [rcir, P4, d1]]
[inter, P1, [nline, P5, P4, a1], [rcir, P5, d1]]
[on, X2, [nline, P4, X0, a1]]
[on, X3, [nline, P4, P5, a1]]
[inter, X4, [pline, P4, P4, P5], [rcir, P4, d3]]
[inter, P2, [pline, X4, P4, X3], [rcir, P1, d2]]
[inter, P3, [nline, P4, P5, a1], [a_cir, P2, P4, a1]]
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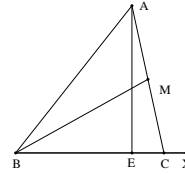
**Example 10** The constraints:  $[\text{dis}, B, C, \text{a1}]$ ,  $[\text{angle}, B, A, C, \text{a2}]$ ,  $[\text{pl\_dis}, B, A, C, \text{a3}]$ .

The construction sequence is  
 [point,  $B$ ,  $X_0$ ]  
 [inter,  $C$ , [line,  $B$ ,  $X_0$ ], [rcir,  $B$ ,  $a_1$ ]]  
 [inter,  $X_1$ , [di\_cir,  $B$ ,  $C$ ], [rcir,  $B$ ,  $a_3$ ]]  
 [inter,  $A$ , [line,  $C$ ,  $X_1$ ], [a\_cir,  $B$ ,  $C$ ,  $a_2$ ]]



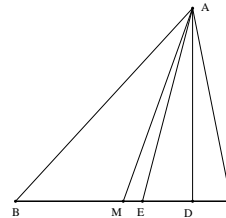
**Example 11** The constraints: [dis,  $B$ ,  $M$ ,  $d_2$ ], [mid,  $M$ ,  $A$ ,  $C$ ], [angle,  $B$ ,  $A$ ,  $C$ ,  $a_1$ ], [pl\_dis,  $A$ ,  $B$ ,  $C$ ,  $d_1$ ].

The construction sequence is  
 [point,  $B$ ,  $X_{17}$ ]  
 [on,  $X_{18}$ , [d\_line,  $X_{17}$ ,  $B$ ,  $d_1$ ]]  
 [on,  $X_{19}$ , [d\_line,  $X_{17}$ ,  $B$ ,  $d_1$ ]]  
 [mid,  $X_{20}$ ,  $X_{18}$ ,  $B$ ]  
 [inter,  $M$ , [pline,  $X_{20}$ ,  $B$ ,  $X_{17}$ ], [rcir,  $B$ ,  $d_2$ ]]  
 [sym,  $X_{22}$ ,  $M$ ,  $B$ ]  
 [sym,  $X_{23}$ ,  $M$ ,  $X_{17}$ ]  
 [inter,  $A$ , [line,  $X_{22}$ ,  $X_{23}$ ], [a\_cir,  $B$ ,  $M$ ,  $a_1$ ]]  
 [sym,  $C$ ,  $M$ ,  $A$ ]



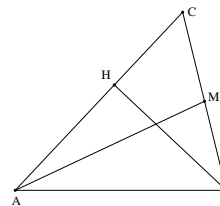
**Example 12** The constraints: [dis,  $A$ ,  $D$ ,  $d_1$ ], [dis,  $A$ ,  $E$ ,  $d_2$ ], [dis,  $A$ ,  $M$ ,  $d_3$ ], [mid,  $M$ ,  $B$ ,  $C$ ], [perp,  $A$ ,  $D$ ,  $B$ ,  $C$ ], [eqangle,  $B$ ,  $A$ ,  $E$ ,  $E$ ,  $A$ ,  $C$ ], [coll,  $B$ ,  $C$ ,  $E$ ], [coll,  $B$ ,  $C$ ,  $D$ ].

The construction sequence is  
 [point,  $A$ ,  $X_0$ ]  
 [inter,  $D$ , [line,  $A$ ,  $X_0$ ], [rcir,  $A$ ,  $d_1$ ]]  
 [inter,  $E$ , [tline,  $D$ ,  $A$ ,  $D$ ], [rcir,  $A$ ,  $d_2$ ]]  
 [inter,  $M$ , [line,  $D$ ,  $E$ ], [rcir,  $A$ ,  $d_3$ ]]  
 [inter,  $X_7$ , [line,  $A$ ,  $E$ ], [tline,  $M$ ,  $M$ ,  $E$ ]]  
 [inter,  $X_8$ , [bline,  $A$ ,  $X_7$ ], [line,  $M$ ,  $X_7$ ]]  
 [inter,  $B$ , [line,  $M$ ,  $D$ ], [pcir,  $X_8$ ,  $A$ ]]  
 [inter,  $C$ , [tline,  $B$ ,  $A$ ,  $D$ ], [pcir,  $M$ ,  $B$ ]]



**Example 13** The constraints: [dis,  $A$ ,  $B$ ,  $d_1$ ], [pl\_dis,  $B$ ,  $A$ ,  $C$ ,  $d_2$ ], [mid,  $M$ ,  $B$ ,  $C$ ], [dis,  $A$ ,  $M$ ,  $d_3$ ].

The construction sequence is  
 [point,  $A$ ,  $X_0$ ]  
 [inter,  $B$ , [line,  $A$ ,  $X_0$ ], [rcir,  $A$ ,  $d_1$ ]]  
 [inter,  $X_1$ , [di\_cir,  $B$ ,  $A$ ], [rcir,  $B$ ,  $d_2$ ]]  
 [on,  $X_2$ , [rcir,  $A$ ,  $d_3$ ]]

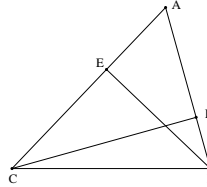


[sym,  $X_3$ ,  $A$ ,  $X_2$ ]  
 [sym,  $X_4$ ,  $A$ ,  $B$ ]  
 [inter,  $C$ , [line,  $A$ ,  $X_1$ ], [dcir,  $X_4$ ,  $X_2$ ,  $X_3$ ]]  
 [mid,  $M$ ,  $B$ ,  $C$ ]

**Example 14** The constraints: [angle,  $C$ ,  $A$ ,  $B$ ,  $a_1$ ], [pl-dis,  $C$ ,  $A$ ,  $B$ ,  $d_1$ ], [pl-dis,  $A$ ,  $B$ ,  $C$ ,  $d_2$ ].

The construction sequence is

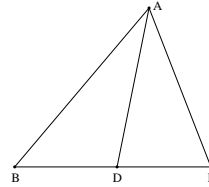
[point,  $A$ ]  
 [point,  $X_4$ ]  
 [inter,  $C$ , [d\_line,  $X_4$ ,  $A$ ,  $d_1$ ], [nline,  $A$ ,  $X_4$ , [r,  $a_1$ ]]]  
 [inter,  $X_5$ , [di\_cir,  $A$ ,  $C$ ], [rcir,  $A$ ,  $d_2$ ]]  
 [inter,  $B$ , [line,  $A$ ,  $X_4$ ], [line,  $C$ ,  $X_5$ ]]



**Example 15** The constraints: [dis,  $B$ ,  $C$ ,  $d_1$ ], [coll,  $D$ ,  $B$ ,  $C$ ], [angle,  $B$ ,  $A$ ,  $D$ ,  $a_1$ ], [eqangle,  $B$ ,  $A$ ,  $D$ ,  $D$ ,  $A$ ,  $C$ ]. The following points and their construction order are given:  $B$ ,  $C$ ,  $D$ .

The construction sequence is

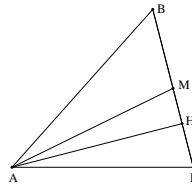
[point,  $B$ ]  
 [on,  $C$ , [rcir,  $B$ ,  $d_1$ ]]  
 [on,  $D$ , [line,  $B$ ,  $C$ ]]  
 [harmonic,  $X_1$ ,  $D$ ,  $B$ ,  $C$ ]  
 [inter,  $A$ , [a\_cir,  $B$ ,  $D$ ,  $a_1$ ], [di\_cir,  $D$ ,  $X_1$ ]]



**Example 16** The constraints: [dis,  $A$ ,  $H$ ,  $d_1$ ], [dis,  $A$ ,  $M$ ,  $d_2$ ], [angle,  $A$ ,  $B$ ,  $C$ ,  $a_1$ ], [mid,  $M$ ,  $B$ ,  $C$ ], [perp,  $H$ ,  $A$ ,  $B$ ,  $C$ ], [coll,  $H$ ,  $B$ ,  $C$ ].

The construction sequence is

[point,  $A$ ,  $X_0$ ]  
 [inter,  $H$ , [line,  $A$ ,  $X_0$ ], [rcir,  $A$ ,  $d_1$ ]]  
 [inter,  $M$ , [tline,  $H$ ,  $H$ ,  $X_0$ ], [rcir,  $A$ ,  $d_2$ ]]  
 [inter,  $B$ , [line,  $M$ ,  $H$ ], [a\_cir,  $A$ ,  $M$ ,  $a_1$ ]]  
 [inter,  $C$ , [tline,  $B$ ,  $H$ ,  $A$ ], [pcir,  $M$ ,  $B$ ]]

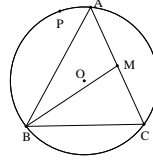


**Example 17** The constraints: [dis,  $B$ ,  $C$ ,  $d_1$ ], [cong,  $O$ ,  $P$ ,  $O$ ,  $A$ ], [cong,  $O$ ,  $P$ ,  $O$ ,  $B$ ], [cong,  $O$ ,  $P$ ,  $O$ ,  $C$ ], [mid,  $M$ ,  $A$ ,  $C$ ], [dis,  $B$ ,  $M$ ,  $d_2$ ]. The following points and their construction order are given:  $O$ ,  $P$ .



The construction sequence is

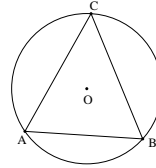
```
[point, O, P, U, V]
[on, B, [pcir, O, P]]
[inter, C, [pcir, O, P], [rcir, B, d1]]
[on, X2, [rcir, B, d2]]
[sym, X3, B, X2]
[sym, X4, B, C]
[inter, A, [dcir, X4, X2, X3], [pcir, O, P]]
[mid, M, A, C]
```



**Example 18** The constraints: [dis, O, B, r], [dis, O, C, r], [dis, O, A, r], [angle, B, A, C, a1], [sq-sum, B, A, C, d1].

The construction sequence is

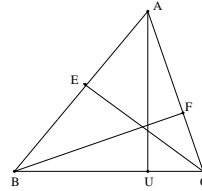
```
[point, O, X0]
[inter, B, [line, O, X0], [rcir, O, r]]
[on, X1, [pcir, O, B]]
[inter, C, [nline, X1, B, a1], [rcir, O, r]]
[inter, X2, [line, B, C], [rcir, B, d1]]
[on, X3, [di_cir, B, X2]]
[inter, X4, [dcir, B, B, X3], [dcir, C, X3, X2]]
[mid, X5, B, C]
[inter, A, [pcir, X5, X4], [rcir, O, r]]
```



**Example 19** The constraints: [dis, B, C, d1], [pl\_dis, A, B, C, d2], [coll, A, E, B], [coll, A, F, C], [perp, B, F, A, C], [perp, C, E, A, B], [sratio, B, F, C, E, r1, r2].

The construction sequence is

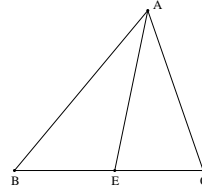
```
[point, B, X0]
[inter, C, [line, B, X0], [rcir, B, d1]]
[mratio, X1, B, C, r1, r2]
[mratio, X2, B, C, [r, r1], r2]
[inter, A, [d_line, B, C, d2], [di_cir, X1, X2]]
[foot, E, C, A, B]
[foot, F, B, A, C]
```



**Example 20** The constraints: [dis, B, C, d1], [coll, E, B, C], [dis, A, E, d1], [eqangle, B, A, E, E, A, C], [sratio, A, B, A, C, r1, r2].

The construction sequence is

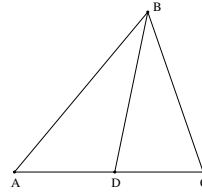
```
[point, B, X0]
[inter, C, [line, B, X0], [rcir, B, d1]]
[mratio, E, B, C, r1, r2]
[mratio, X1, B, C, r1, r2]
[mratio, X2, B, C, [r, r1], r2]
[inter, A, [di_cir, X1, X2], [rcir, E, d1]]
```



**Example 21** The constraints: [dis, A, D, d1], [dis, C, D, d2], [coll, D, A, C], [eqangle, A, B, D, D, B, C], [dis, B, D, d3].

The construction sequence is

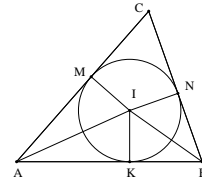
```
[point, A, X0]
[inter, D, [line, A, X0], [rcir, A, d1]]
[inter, C, [line, D, A], [rcir, D, d2]]
[harmonic, X1, D, A, C]
[inter, B, [di_cir, D, X1], [rcir, D, d3]]
```



**Example 22** The constraints: [dis, I, M, d1], [dis, I, N, d1], [dis, I, K, d1], [coll, B, X, C], [sratio, B, C, B, A, r1, r2], [angle, A, B, C, a1], [coll, M, A, B], [perp, I, M, A, B], [coll, N, B, C], [perp, I, N, B, C], [coll, K, A, C], [perp, I, K, A, C]. The following points and their construction order are given: B, X.

The construction sequence is

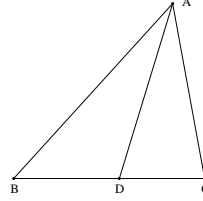
```
[point, B, X]
[on, X0, [nline, B, X, [r, a1]]]
[inter, X2, [tline, B, B, X0], [rcir, B, d1]]
[inter, I, [d_line, B, X, d1], [pline, X2, B, X1]]
[on, X3, [nline, B, I, [div, a1, 2]]]
[foot, M, I, B, X3]
[foot, N, I, X, B]
[inter, K, [tline, I, X8, X9], [rcir, I, d1]]
[inter, C, [line, B, X], [tline, K, I, K]]
[inter, A, [line, K, C], [nline, B, C, a1]]
```



**Example 23** The constraints: [dis, B, C, d1], [coll, D, B, C], [angle, B, A, C, a1], [eqangle, B, A, D, D, A, C]. The following points and their construction order are given: B, C, D.

The construction sequence is

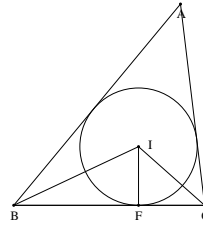
[point,  $B$ ]  
 [on,  $C$ , [rcir,  $B$ ,  $d1$ ]]  
 [on,  $D$ , [line,  $B$ ,  $C$ ]]  
 [harmonic,  $X_1$ ,  $D$ ,  $B$ ,  $C$ ]  
 [inter,  $A$ , [a\_cir,  $B$ ,  $C$ ,  $a1$ ], [di\_cir,  $D$ ,  $X_1$ ]]



**Example 24** The constraints: [dis,  $B$ ,  $C$ ,  $d1$ ], [dis,  $I$ ,  $F$ ,  $d2$ ], [angle,  $B$ ,  $A$ ,  $C$ ,  $a1$ ], [coll,  $F$ ,  $B$ ,  $C$ ], [perp,  $I$ ,  $F$ ,  $B$ ,  $C$ ], [eqangle,  $B$ ,  $A$ ,  $I$ ,  $I$ ,  $A$ ,  $C$ ], [eqangle,  $A$ ,  $B$ ,  $I$ ,  $I$ ,  $B$ ,  $C$ ]. The following points and their construction order are given:  $B$ ,  $C$ .

The construction sequence is

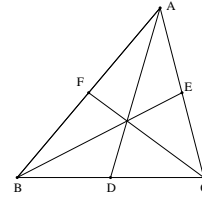
[point,  $B$ ]  
 [on,  $C$ , [rcir,  $B$ ,  $d1$ ]]  
 [on,  $X_1$ , [nline,  $B$ ,  $C$ ,  $a1$ ]]  
 [inter,  $X_2$ , [bline,  $B$ ,  $C$ ], [tline,  $B$ ,  $B$ ,  $X_1$ ]]  
 [mid,  $X_3$ ,  $B$ ,  $C$ ]  
 [inter,  $X_4$ , [bline,  $B$ ,  $C$ ], [pcir,  $X_2$ ,  $B$ ]]  
 [inter,  $I$ , [d\_line,  $B$ ,  $C$ ,  $d2$ ], [line,  $X_3$ ,  $X_4$ ]]  
 [foot,  $F$ ,  $I$ ,  $B$ ,  $C$ ]  
 [inter,  $A$ , [a\_cir,  $B$ ,  $C$ ,  $a1$ ], [a\_cir,  $I$ ,  $C$ , [d,  $a1$ , 2]]]



**Example 25** The constraints: [dis,  $A$ ,  $D$ ,  $d1$ ], [mid,  $E$ ,  $A$ ,  $C$ ], [mid,  $D$ ,  $B$ ,  $C$ ], [dis,  $B$ ,  $E$ ,  $d2$ ], [dis,  $C$ ,  $F$ ,  $d3$ ], [mid,  $F$ ,  $A$ ,  $B$ ].

The construction sequence is

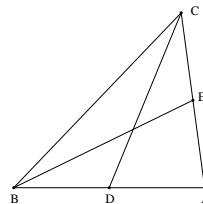
[point,  $A$ ,  $X_0$ ]  
 [inter,  $D$ , [line,  $A$ ,  $X_0$ ], [rcir,  $A$ ,  $d1$ ]]  
 [mratio,  $X_1$ ,  $A$ ,  $D$ , 2, 1]  
 [inter,  $X_2$ , [rcir,  $A$ , [div, [mul,  $d3$ , 2], 3]], [rcir,  $X_1$ , [div, [mul,  $d2$ , 2], 3]]]  
 [mid,  $E$ ,  $X_1$ ,  $X_2$ ]  
 [sym,  $C$ ,  $E$ ,  $A$ ]  
 [sym,  $B$ ,  $D$ ,  $C$ ]  
 [mid,  $F$ ,  $A$ ,  $B$ ]



**Example 26** The constraints: [dis,  $B$ ,  $D$ ,  $d1$ ], [mid,  $E$ ,  $A$ ,  $B$ ], [mid,  $D$ ,  $A$ ,  $C$ ], [dis,  $C$ ,  $E$ ,  $d2$ ], [angle,  $B$ ,  $A$ ,  $C$ ,  $a1$ ].

The construction sequence is

[point,  $B$ ,  $X_0$ ]  
 [inter,  $D$ , [line,  $B$ ,  $X_0$ ], [rcir,  $B$ ,  $d1$ ]]



```

[mratio, X1, B, D, 2, 1]
[on, X2, [nline, B, D, a1]]
[inter, X3, [bline, B, D], [tline, B, B, X2]]
[mid, X4, X3, D]
[mid, X5, X3, B]
[inter, E, [dcir, X5, X3, X4], [rcir, X1, [div, d2, 3]]]
[sym, A, E, B]
[sym, C, D, A]

```

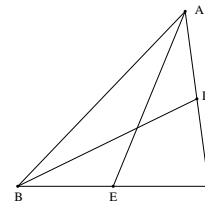
**Example 27** The constraints: [dis, B, D, d1], [dis, A, E, d2], [mid, E, C, B], [mid, D, A, C], [angle, B, A, C, a1].

The construction sequence is

```

[point, B, X0]
[inter, D, [line, B, X0], [rcir, B, d1]]
[mratio, X1, B, D, 2, 1]
[inter, A, [a_cir, B, D, a1], [rcir, D, [div, [mul, d2, 2], 3]]]
[sym, C, D, A]
[mid, E, C, B]

```



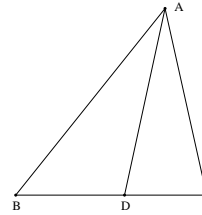
**Example 28** The constraints: [dis, A, D, d1], [dis, A, B, d2], [dis, A, C, d3], [eqangle, B, A, D, D, A, C], [coll, B, C, D].

The construction sequence is

```

[point, A, X0]
[inter, D, [line, A, X0], [rcir, A, d1]]
[on, X1, [rcir, A, d3]]
[lratio, X2, D, A, d2, d3]
[lratio, X3, A, X1, d2, d3]
[inter, B, [dcir, X2, A, X3], [rcir, A, d2]]
[inter, C, [aline, A, D, D, A, B], [line, B, D]]

```



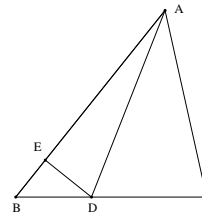
**Example 29** The constraints: [perp, A, B, D, E], [coll, A, B, E], [dis, D, E, d1], [dis, A, B, d2], [dis, A, C, d3], [sratio, B, D, D, C, r1, r2], [coll, B, C, D].

The construction sequence is

```

[point, A, X0]
[inter, B, [line, A, X0], [rcir, A, d2]]
[on, X3, [d_line, A, B, d1]]
[on, X4, [d_line, A, B, d1]]
[lratio, X5, X3, B, r1, r2]
[inter, C, [pline, X5, X3, X4], [rcir, A, d3]]

```

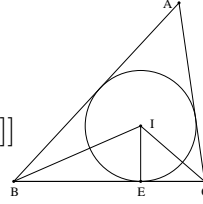


[inter,  $D$ , [d\_line,  $A, B, d1$ ], [line,  $B, C$ ]]  
 [foot,  $E, D, A, B$ ]

**Example 30** The constraints: [dis,  $B, C, d1$ ], [dis,  $I, E, d2$ ], [angle,  $C, B, A, a1$ ], [coll,  $E, B, C$ ], [perp,  $I, E, B, C$ ], [eqangle,  $A, B, I, I, B, C$ ], [eqangle,  $A, C, I, I, C, B$ ].

The construction sequence is

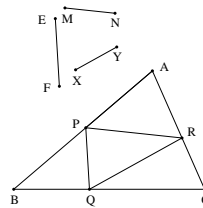
[point,  $B, X_0$ ]  
 [inter,  $C$ , [line,  $B, X_0$ ], [rcir,  $B, d1$ ]]  
 [inter,  $I$ , [d\_line,  $C, X_0, d2$ ], [nline,  $B, X_0, [d, a1, 2]]$ ]  
 [foot,  $E, I, B, C$ ]  
 [inter,  $A$ , [aline,  $C, I, I, C, B$ ], [nline,  $B, C, a1$ ]]



**Example 31** The constraints: [coll,  $R, A, C$ ], [coll,  $P, A, B$ ], [coll,  $Q, B, C$ ], [para,  $P, Q, E, F$ ], [para,  $P, R, M, N$ ], [para,  $Q, R, X, Y$ ]. The following points and their construction order are given:  $A, B, C, E, F, M, N, X, Y$ .

The construction sequence is

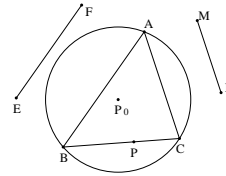
[point,  $A, B, C, E, F, M, N, X, Y$ ]  
 [inter,  $X_0$ , [line,  $B, C$ ], [pline,  $A, E, F$ ]]  
 [inter,  $X_1$ , [pline,  $A, M, N$ ], [pline,  $X_0, X, Y$ ]]  
 [inter,  $R$ , [line,  $A, C$ ], [line,  $B, X_1$ ]]  
 [inter,  $P$ , [line,  $A, B$ ], [pline,  $R, M, N$ ]]  
 [inter,  $Q$ , [line,  $B, C$ ], [pline,  $R, X, Y$ ]]



**Example 32** The constraints: [dis,  $B, O, d1$ ], [dis,  $C, O, d1$ ], [dis,  $A, O, d1$ ], [para,  $A, B, E, F$ ], [para,  $A, C, M, N$ ], [coll,  $P, B, C$ ]. The following points and their construction order are given:  $O, E, F, M, N, P$ .

The construction sequence is

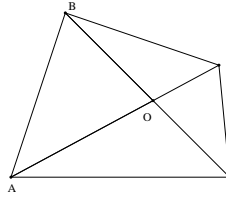
[point,  $O, E, F, M, N, P$ ]  
 [on,  $X_1$ , [rcir,  $O, d1$ ]]  
 [inter,  $X_2$ , [pline,  $X_1, E, F$ ], [pcir,  $O, X_1$ ]]  
 [inter,  $X_3$ , [pline,  $X_1, M, N$ ], [pcir,  $O, X_1$ ]]  
 [mid,  $X_4, X_2, X_3$ ]  
 [inter,  $B$ , [ta\_line,  $P, O, X_4$ ], [rcir,  $O, d1$ ]]  
 [inter,  $C$ , [line,  $P, B$ ], [rcir,  $O, d1$ ]]  
 [inter,  $A$ , [pline,  $B, E, F$ ], [pline,  $C, M, N$ ]]



**Example 33** The constraints: [angle,  $D, A, B, a1$ ], [dis,  $A, D, d1$ ], [dis,  $A, B, d2$ ], [coll,  $O, A, C$ ], [coll,  $O, B, D$ ], [angle,  $A, O, D, a2$ ], [dis,  $A, C, d3$ ].

The construction sequence is

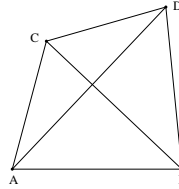
```
[point, D, X0]
[inter, A, [line, D, X0], [rcir, D, d1]]
[inter, B, [nline, A, D, a1], [rcir, A, d2]]
[inter, O, [line, B, D], [a_cir, A, D, a2]]
[inter, C, [line, O, A], [rcir, A, d3]]
```



**Example 34** The constraints: [dis, A, B, d1], [dis, B, C, d2], [dis, C, D, d3], [angle, B, A, D, a1], [angle, A, D, C, a2].

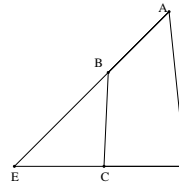
The construction sequence is

```
[point, A, X0]
[inter, B, [line, A, X0], [rcir, A, d1]]
[on, X2, [nline, A, X0, a1]]
[on, X3, [nline, A, X2, [r, a2]]]
[on, X4, [nline, A, B, a1]]
[inter, X5, [pline, A, A, X3], [rcir, A, d3]]
[inter, C, [pline, X5, A, X4], [rcir, B, d2]]
[inter, D, [nline, A, B, a1], [a_cir, A, C, a2]]
```



**Example 35** The constraints: [dis, A, B, d1], [dis, B, C, d2], [dis, C, D, d3], [dis, D, A, d4], [coll, E, A, B], [coll, E, C, D], [angle, A, E, C, a1].

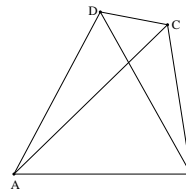
```
[point, A, X0]
[inter, B, [line, A, X0], [rcir, A, d1]]
[on, X2, [nline, A, X0, a1]]
[inter, X3, [pline, A, A, X2], [rcir, A, d3]]
[inter, C, [rcir, B, d2], [rcir, X3, d4]]
[inter, E, [line, A, B], [pline, C, X2, A]]
[inter, D, [line, E, C], [pline, A, C, X3]]
```



**Example 36** The constraints: [dis, A, B, d1], [dis, D, C, d2], [angle, B, A, C, a1], [angle, A, C, D, a2], [angle, A, D, B, a3].

The construction sequence is

```
[point, A, X0]
[inter, B, [line, A, X0], [rcir, A, d1]]
[on, X1, [nline, A, X0, a1]]
[on, X2, [nline, A, X1, [r, a2]]]
[on, X3, [nline, A, B, a1]]
[inter, X4, [pline, A, A, X2], [rcir, A, d2]]
```

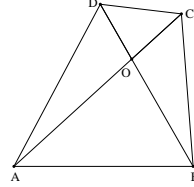


[inter,  $D$ , [pline,  $X_4$ ,  $A$ ,  $X_3$ ], [a\_cir,  $A$ ,  $B$ ,  $a_3$ ]]  
 [inter,  $C$ , [nline,  $A$ ,  $B$ ,  $a_1$ ], [a\_cir,  $A$ ,  $D$ ,  $a_2$ ]]

**Example 37** The constraints: [dis,  $B$ ,  $D$ ,  $d_1$ ], [dis,  $A$ ,  $C$ ,  $d_2$ ], [angle,  $B$ ,  $C$ ,  $D$ ,  $a_1$ ], [sratio,  $A$ ,  $B$ ,  $A$ ,  $D$ ,  $r_1$ ,  $r_2$ ], [coll,  $O$ ,  $A$ ,  $C$ ], [coll,  $O$ ,  $B$ ,  $D$ ], [angle,  $A$ ,  $O$ ,  $B$ ,  $a_2$ ].

The construction sequence is

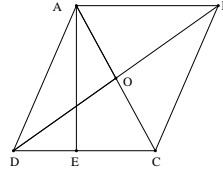
[point,  $B$ ,  $X_0$ ]  
 [inter,  $D$ , [line,  $B$ ,  $X_0$ ], [rcir,  $B$ ,  $d_1$ ]]  
 [on,  $X_1$ , [nline,  $B$ ,  $X_0$ ,  $a_2$ ]]  
 [on,  $X_2$ , [nline,  $B$ ,  $D$ ,  $a_1$ ]]  
 [inter,  $X_3$ , [pline,  $B$ ,  $D$ ], [tline,  $B$ ,  $B$ ,  $X_2$ ]]  
 [inter,  $X_4$ , [pline,  $X_3$ ,  $B$ ,  $X_1$ ], [rcir,  $X_3$ ,  $d_2$ ]]  
 [mratio,  $X_5$ ,  $B$ ,  $D$ ,  $r_1$ ,  $r_2$ ]  
 [mratio,  $X_6$ ,  $B$ ,  $D$ , [r,  $r_1$ ],  $r_2$ ]  
 [inter,  $A$ , [dcir,  $X_4$ ,  $X_3$ ,  $B$ ], [di\_cir,  $X_5$ ,  $X_6$ ]]  
 [inter,  $O$ , [line,  $B$ ,  $D$ ], [pline,  $A$ ,  $X_1$ ,  $B$ ]]  
 [inter,  $C$ , [line,  $O$ ,  $A$ ], [a\_cir,  $B$ ,  $D$ ,  $a_1$ ]]



**Example 38** The constraints: [dis,  $A$ ,  $C$ ,  $a_1$ ], [dis,  $C$ ,  $E$ ,  $a_2$ ], [coll,  $E$ ,  $A$ ,  $D$ ], [perp,  $A$ ,  $D$ ,  $E$ ,  $C$ ], [cong,  $D$ ,  $A$ ,  $D$ ,  $C$ ], [para,  $A$ ,  $D$ ,  $B$ ,  $C$ ], [para,  $D$ ,  $C$ ,  $A$ ,  $B$ ].

The construction sequence is

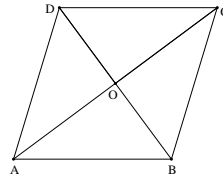
[point,  $A$ ,  $X_0$ ]  
 [inter,  $C$ , [line,  $A$ ,  $X_0$ ], [rcir,  $A$ ,  $a_1$ ]]  
 [inter,  $E$ , [di\_cir,  $A$ ,  $C$ ], [rcir,  $C$ ,  $a_2$ ]]  
 [inter,  $D$ , [pline,  $A$ ,  $C$ ], [line,  $E$ ,  $A$ ]]  
 [inter,  $B$ , [pline,  $A$ ,  $D$ ,  $C$ ], [pline,  $C$ ,  $A$ ,  $D$ ]]



**Example 39** The constraints: [dis,  $A$ ,  $B$ ,  $d_1$ ], [dis,  $A$ ,  $D$ ,  $d_2$ ], [para,  $A$ ,  $B$ ,  $C$ ,  $D$ ], [para,  $A$ ,  $D$ ,  $B$ ,  $C$ ], [coll,  $O$ ,  $A$ ,  $C$ ], [coll,  $O$ ,  $B$ ,  $D$ ], [angle,  $A$ ,  $O$ ,  $B$ ,  $a_1$ ].

The construction sequence is

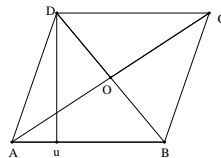
[point,  $A$ ,  $X_0$ ]  
 [inter,  $B$ , [line,  $A$ ,  $X_0$ ], [rcir,  $A$ ,  $d_1$ ]]  
 [on,  $X_{15}$ , [rcir,  $A$ ,  $d_2$ ]]  
 [mid,  $X_{16}$ ,  $A$ ,  $X_{15}$ ]  
 [mid,  $X_{17}$ ,  $A$ ,  $B$ ]  
 [inter,  $O$ , [a\_cir,  $A$ ,  $B$ ,  $a_1$ ], [dcir,  $X_{17}$ ,  $A$ ,  $X_{16}$ ]]  
 [sym,  $D$ ,  $O$ ,  $B$ ]  
 [inter,  $C$ , [line,  $O$ ,  $A$ ], [pline,  $D$ ,  $A$ ,  $B$ ]]



**Example 40** The constraints: [coll,  $O$ ,  $A$ ,  $C$ ], [coll,  $O$ ,  $B$ ,  $D$ ], [angle,  $A$ ,  $O$ ,  $B$ ,  $a_1$ ], [para,  $A$ ,  $B$ ,  $C$ ,  $D$ ], [para,  $A$ ,  $D$ ,  $B$ ,  $C$ ], [pl\_dis,  $D$ ,  $A$ ,  $B$ ,  $d_1$ ]. The following points and their construction order are given:  $A$ ,  $B$ .

The construction sequence is

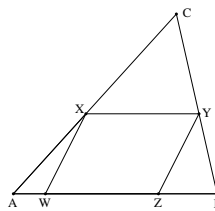
[point,  $A$ ,  $B$ ]  
 [on,  $X_2$ , [d\_line,  $A$ ,  $B$ ,  $d_1$ ]]  
 [on,  $X_3$ , [d\_line,  $A$ ,  $B$ ,  $d_1$ ]]  
 [inter,  $O$ , [m\_line,  $B$ ,  $X_2$ ,  $X_3$ ], [a\_cir,  $A$ ,  $B$ ,  $a_1$ ]]  
 [sym,  $C$ ,  $O$ ,  $A$ ]  
 [inter,  $D$ , [line,  $O$ ,  $B$ ], [pline,  $C$ ,  $A$ ,  $B$ ]]



**Example 41** The constraints: [sratio,  $Y$ ,  $Z$ ,  $Y$ ,  $X$ ,  $r_1$ ,  $r_2$ ], [coll,  $W$ ,  $A$ ,  $B$ ], [coll,  $Z$ ,  $A$ ,  $B$ ], [coll,  $X$ ,  $A$ ,  $C$ ], [coll,  $Y$ ,  $C$ ,  $B$ ], [para,  $X$ ,  $Y$ ,  $A$ ,  $B$ ], [para,  $X$ ,  $W$ ,  $Y$ ,  $Z$ ], [angle,  $X$ ,  $W$ ,  $B$ ,  $a_1$ ]. The following points and their construction order are given:  $A$ ,  $B$ ,  $C$ .

The construction sequence is

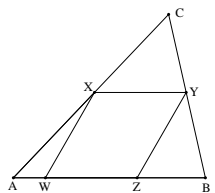
[point,  $A$ ,  $B$ ,  $C$ ]  
 [inter,  $X_0$ , [nline,  $A$ ,  $B$ ,  $a_1$ ], [rcir,  $A$ ,  $r_1$ ]]  
 [inter,  $X_1$ , [line,  $A$ ,  $B$ ], [rcir,  $A$ ,  $r_2$ ]]  
 [inter,  $X_2$ , [pline,  $X_0$ ,  $A$ ,  $B$ ], [pline,  $X_1$ ,  $A$ ,  $C$ ]]  
 [inter,  $Y$ , [line,  $A$ ,  $X_2$ ], [line,  $C$ ,  $B$ ]]  
 [inter,  $Z$ , [line,  $A$ ,  $B$ ], [pline,  $Y$ ,  $A$ ,  $X_0$ ]]  
 [inter,  $X$ , [line,  $A$ ,  $C$ ], [pline,  $Y$ ,  $A$ ,  $B$ ]]  
 [inter,  $W$ , [line,  $A$ ,  $B$ ], [pline,  $X$ ,  $X_0$ ,  $A$ ]]



**Example 42** The constraints: [coll,  $Y$ ,  $B$ ,  $C$ ], [coll,  $X$ ,  $A$ ,  $C$ ], [coll,  $W$ ,  $A$ ,  $B$ ], [coll,  $Z$ ,  $A$ ,  $B$ ], [cong,  $Y$ ,  $X$ ,  $Y$ ,  $Z$ ], [angle,  $Y$ ,  $Z$ ,  $W$ ,  $a_1$ ], [para,  $W$ ,  $X$ ,  $Y$ ,  $Z$ ], [para,  $X$ ,  $Y$ ,  $A$ ,  $B$ ]. The following points and their construction order are given:  $A$ ,  $B$ ,  $C$ .

The construction sequence is

[point,  $A$ ,  $B$ ,  $C$ ]  
 [inter,  $X_0$ , [line,  $A$ ,  $B$ ], [rcir,  $A$ , 10]]  
 [inter,  $X_1$ , [nline,  $A$ ,  $B$ ,  $a_1$ ], [rcir,  $A$ , 10]]  
 [inter,  $X_2$ , [pline,  $X_0$ ,  $A$ ,  $C$ ], [pline,  $X_1$ ,  $A$ ,  $B$ ]]  
 [inter,  $Y$ , [line,  $A$ ,  $X_2$ ], [line,  $B$ ,  $C$ ]]  
 [inter,  $X$ , [line,  $A$ ,  $C$ ], [pline,  $Y$ ,  $A$ ,  $B$ ]]  
 [inter,  $W$ , [line,  $A$ ,  $B$ ], [pline,  $X$ ,  $A$ ,  $X_1$ ]]  
 [mid,  $X_5$ ,  $W$ ,  $Y$ ]  
 [inter,  $Z$ , [pline,  $Y$ ,  $X_1$ ,  $A$ ], [tline,  $X_5$ ,  $W$ ,  $Y$ ]]

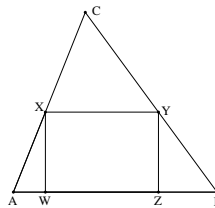




**Example 43** The constraints: [coll,  $Y, B, C$ ], [coll,  $X, A, C$ ], [coll,  $W, A, B$ ], [coll,  $Z, A, B$ ], [sratio,  $Y, X, Y, Z, r1, r2$ ], [perp,  $X, Y, Y, Z$ ], [para,  $W, X, Y, Z$ ], [para,  $X, Y, A, B$ ]. The following points and their construction order are given:  $A, B, C$ .

The construction sequence is

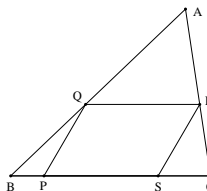
[point,  $A, B, C$ ]  
 [inter,  $X_0$ , [line,  $A, B$ ], [rcir,  $A, r1$ ]]  
 [inter,  $X_1$ , [d\_line,  $A, B, r2$ ], [pline,  $X_0, A, C$ ]]  
 [inter,  $Y$ , [line,  $A, X_1$ ], [line,  $B, C$ ]]  
 [inter,  $X$ , [line,  $A, C$ ], [pline,  $Y, A, B$ ]]  
 [inter,  $W$ , [line,  $A, B$ ], [tline,  $X, X, Y$ ]]  
 [inter,  $Z$ , [line,  $A, B$ ], [tline,  $Y, X, Y$ ]]



**Example 44** The constraints: [coll,  $P, B, C$ ], [coll,  $Q, A, B$ ], [coll,  $R, A, C$ ], [coll,  $S, B, C$ ], [para,  $Q, P, R, S$ ], [para,  $Q, R, P, S$ ], [sratio,  $Q, P, Q, R, r1, r2$ ]. The following points and their construction order are given:  $A, B, C, P$ .

The construction sequence is

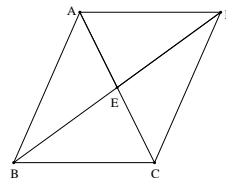
[point,  $A, B, C$ ]  
 [on,  $P$ , [line,  $B, C$ ]]  
 [lratio,  $X_0, B, C, r1, r2$ ]  
 [inter,  $X_1$ , [line,  $A, P$ ], [pcir,  $B, X_0$ ]]  
 [inter,  $Q$ , [line,  $A, B$ ], [pline,  $P, B, X_1$ ]]  
 [inter,  $R$ , [line,  $A, C$ ], [pline,  $Q, B, P$ ]]  
 [inter,  $S$ , [line,  $B, C$ ], [pline,  $R, Q, P$ ]]



**Example 45** The constraints: [dis,  $A, C, d1$ ], [angle,  $A, B, C, a1$ ], [angle,  $B, E, C, a2$ ], [para,  $A, B, C, D$ ], [para,  $A, D, B, C$ ], [coll,  $E, A, C$ ], [coll,  $E, B, D$ ].

The construction sequence is

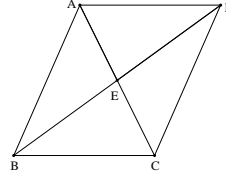
[point,  $A, X_0$ ]  
 [inter,  $C$ , [line,  $A, X_0$ ], [rcir,  $A, d1$ ]]  
 [mid,  $E, A, C$ ]  
 [inter,  $B$ , [nline,  $E, C, a2$ ], [a\_cir,  $A, C, a1$ ]]  
 [inter,  $D$ , [pline,  $A, B, C$ ], [pline,  $C, A, B$ ]]



**Example 46** The constraints: [dis,  $A, B, d1$ ], [angle,  $A, E, B, a1$ ], [angle,  $A, B, C, a2$ ], [para,  $A, B, C, D$ ], [para,  $A, D, B, C$ ], [coll,  $E, A, C$ ], [coll,  $E, B, D$ ].

The construction sequence is

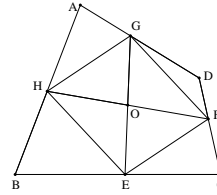
[point, A, X<sub>0</sub>]  
 [inter, B, [line, A, X<sub>0</sub>], [rcir, A, d1]]  
 [on, X<sub>2</sub>, [nline, B, A, a2]]  
 [inter, E, [m\_line, A, B, X<sub>2</sub>], [a\_cir, A, B, a1]]  
 [sym, C, E, A]  
 [inter, D, [line, E, B], [pline, C, A, B]]



**Example 47** The constraints: [para, G, H, E, F], [para, H, E, G, F], [coll, O, G, E], [coll, O, H, F], [coll, A, D, G], [coll, D, C, F], [coll, C, B, E], [coll, B, A, H]. The following points and their construction order are given: A, B, C, D, O.

The construction sequence is

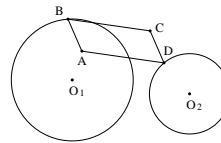
[point, A, B, C, D, O]  
 [sym, X<sub>0</sub>, O, C]  
 [sym, X<sub>1</sub>, O, B]  
 [inter, G, [line, A, D], [line, X<sub>0</sub>, X<sub>1</sub>]]  
 [sym, X<sub>2</sub>, O, D]  
 [sym, X<sub>3</sub>, O, C]  
 [inter, H, [line, B, A], [line, X<sub>2</sub>, X<sub>3</sub>]]  
 [sym, E, O, G]  
 [inter, F, [line, O, H], [pline, G, H, E]]



**Example 48** The constraints: [dis, O<sub>1</sub>, B, d1], [dis, O<sub>2</sub>, D, d2], [para, A, B, C, D], [para, A, D, B, C]. The following points and their construction order are given: A, C, O<sub>1</sub>, O<sub>2</sub>.

The construction sequence is

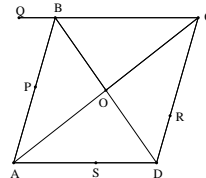
[point, A, C, O<sub>1</sub>, O<sub>2</sub>]  
 [mid, X<sub>1</sub>, A, C]  
 [on, X<sub>2</sub>, [rcir, O<sub>2</sub>, d2]]  
 [sym, X<sub>3</sub>, X<sub>1</sub>, O<sub>2</sub>]  
 [inter, B, [dcir, X<sub>3</sub>, O<sub>2</sub>, X<sub>2</sub>], [rcir, O<sub>1</sub>, d1]]  
 [inter, D, [pline, A, B, C], [pline, C, A, B]]



**Example 49** The constraints: [para, A, B, C, D], [para, A, D, B, C], [coll, O, A, C], [coll, O, B, D], [coll, P, A, B], [coll, Q, B, C], [coll, R, C, D], [coll, S, A, D]. The following points and their construction order are given: O, P, Q, S, R.

The construction sequence is

[point, O, P, Q, S, R]  
 [sym, X<sub>0</sub>, O, R]

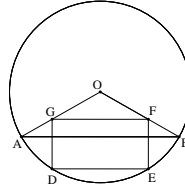


[sym,  $X_1$ ,  $O$ ,  $Q$ ]  
[inter,  $A$ , [line,  $P$ ,  $X_0$ ], [line,  $S$ ,  $X_1$ ]]  
[inter,  $B$ , [line,  $P$ ,  $A$ ], [pline,  $Q$ ,  $S$ ,  $A$ ]]  
[sym,  $C$ ,  $O$ ,  $A$ ]  
[inter,  $D$ , [line,  $O$ ,  $B$ ], [line,  $S$ ,  $A$ ]]

**Example 50** The constraints: [para,  $D$ ,  $E$ ,  $G$ ,  $F$ ], [para,  $D$ ,  $G$ ,  $E$ ,  $F$ ], [perp,  $D$ ,  $G$ ,  $G$ ,  $F$ ], [dis,  $O$ ,  $D$ , d1], [dis,  $O$ ,  $E$ , d1], [dis,  $O$ ,  $A$ , d1], [dis,  $O$ ,  $B$ , d1], [coll,  $O$ ,  $G$ ,  $A$ ], [coll,  $O$ ,  $F$ ,  $B$ ], [sratio,  $G$ ,  $D$ ,  $G$ ,  $F$ ,  $r_1$ ,  $r_2$ ]. The following points and their construction order are given:  $O$ ,  $A$ ,  $B$ .

The construction sequence is

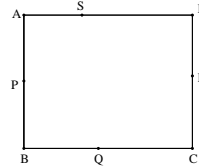
[point,  $O$ ]  
[on,  $A$ , [rcir,  $O$ , d1]]  
[on,  $B$ , [rcir,  $O$ , d1]]  
[lratio,  $X_2$ ,  $A$ ,  $B$ ,  $r_1$ ,  $r_2$ ]  
[inter,  $X_3$ , [tline,  $A$ ,  $A$ ,  $B$ ], [pcir,  $A$ ,  $B$ ]]  
[inter,  $D$ , [line,  $O$ ,  $X_3$ ], [rcir,  $O$ , d1]]  
[inter,  $E$ , [pline,  $D$ ,  $A$ ,  $B$ ], [rcir,  $O$ , d1]]  
[inter,  $G$ , [line,  $O$ ,  $A$ ], [tline,  $D$ ,  $D$ ,  $E$ ]]  
[inter,  $F$ , [line,  $O$ ,  $B$ ], [tline,  $G$ ,  $D$ ,  $G$ ]]



**Example 51** The constraints: [dis,  $B$ ,  $C$ , d1], [para,  $A$ ,  $B$ ,  $C$ ,  $D$ ], [para,  $A$ ,  $D$ ,  $B$ ,  $C$ ], [perp,  $A$ ,  $D$ ,  $D$ ,  $C$ ], [coll,  $P$ ,  $A$ ,  $B$ ], [coll,  $Q$ ,  $B$ ,  $C$ ], [coll,  $R$ ,  $C$ ,  $D$ ], [coll,  $S$ ,  $A$ ,  $D$ ]. The following points and their construction order are given:  $P$ ,  $Q$ ,  $R$ ,  $S$ .

The construction sequence is

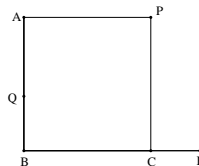
[point,  $P$ ,  $Q$ ,  $R$ ,  $S$ ]  
[mid,  $X_1$ ,  $Q$ ,  $P$ ]  
[mid,  $X_2$ ,  $Q$ ,  $R$ ]  
[inter,  $X_3$ , [di\_cir,  $X_1$ ,  $X_2$ ], [rcir,  $X_1$ , [div, d1, 2]]]  
[inter,  $B$ , [pline,  $Q$ ,  $X_1$ ,  $X_3$ ], [di\_cir,  $P$ ,  $Q$ ]]  
[inter,  $C$ , [line,  $Q$ ,  $B$ ], [pline,  $R$ ,  $P$ ,  $B$ ]]  
[inter,  $A$ , [line,  $P$ ,  $B$ ], [pline,  $S$ ,  $Q$ ,  $B$ ]]  
[inter,  $D$ , [line,  $R$ ,  $C$ ], [line,  $S$ ,  $A$ ]]



**Example 52** The constraints: [cong,  $A$ ,  $B$ ,  $A$ ,  $P$ ], [perp,  $A$ ,  $P$ ,  $P$ ,  $C$ ], [para,  $A$ ,  $B$ ,  $C$ ,  $P$ ], [para,  $A$ ,  $P$ ,  $B$ ,  $C$ ], [coll,  $Q$ ,  $A$ ,  $B$ ], [coll,  $K$ ,  $B$ ,  $C$ ]. The following points and their construction order are given:  $P$ ,  $K$ ,  $Q$ .

The construction sequence is

[point,  $P$ ,  $K$ ,  $Q$ ]  
[inter,  $B$ , [a\_cir,  $P$ ,  $Q$ , [div, pi, 2]], [di\_cir,  $Q$ ,  $K$ ]]

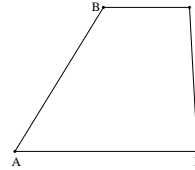


[inter, A, [bline, B, P], [line, Q, B]]  
 [foot, C, P, B, K]

**Example 53** The constraints: [dis, B, C, d1], [dis, C, D, d2], [dis, D, A, d3], [dis, A, B, d4], [para, A, D, B, C].

The construction sequence is

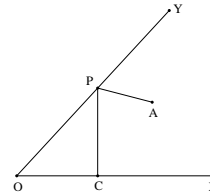
[point, B, X<sub>0</sub>]  
 [inter, C, [line, B, X<sub>0</sub>], [rcir, B, d1]]  
 [inter, X<sub>2</sub>, [pline, B, B, C], [rcir, B, d3]]  
 [inter, D, [rcir, C, d2], [rcir, X<sub>2</sub>, d4]]  
 [inter, A, [pline, D, B, C], [rcir, B, d4]]



**Example 54** The constraints: [coll, P, O, Y], [coll, C, O, X], [perp, P, C, O, X], [sratio, P, A, P, C, r1, r2]. The following points and their construction order are given: O, X, Y, A.

The construction sequence is

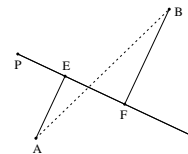
[point, O, X, Y, A]  
 [foot, X<sub>0</sub>, Y, O, X]  
 [lratio, X<sub>1</sub>, Y, X<sub>0</sub>, r<sub>1</sub>, r<sub>2</sub>]  
 [inter, X<sub>2</sub>, [line, O, A], [pcir, Y, X<sub>1</sub>]]  
 [inter, P, [line, O, Y], [pline, A, Y, X<sub>2</sub>]]  
 [foot, C, P, O, X]



**Example 55** The constraints: [coll, E, F, P], [perp, P, E, A, E], [perp, P, F, B, F], [sratio, A, E, B, F, r1, r2]. The following points and their construction order are given: P, A, B.

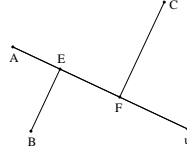
The construction sequence is

[point, P, A, B]  
 [mratio, X<sub>0</sub>, A, B, r1, r2]  
 [foot, E, A, P, X<sub>0</sub>]  
 [foot, F, B, P, E]



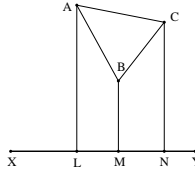
**Example 56** The constraints: [coll, E, F, A], [perp, B, E, A, E], [perp, C, F, A, F], [sratio, A, E, A, F, r1, r2]. The following points and their construction order are given: A, B, C.

The construction sequence is  
[point, A, B, C]  
[mid, X<sub>1</sub>, C, A]  
[lratio, X<sub>4</sub>, A, X<sub>1</sub>, r<sub>1</sub>, r<sub>2</sub>]  
[inter, E, [di\_cir, B, A], [pcir, X<sub>4</sub>, A]]  
[foot, F, C, A, E]



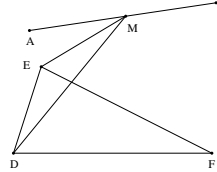
**Example 57** The constraints: [coll, L, M, N], [perp, M, B, M, N], [perp, N, C, N, M], [perp, L, A, L, M], [sratio, B, M, A, L, r<sub>1</sub>, r<sub>2</sub>], [sratio, A, L, C, N, r<sub>2</sub>, r<sub>3</sub>]. The following points and their construction order are given: A, B, C.

The construction sequence is  
[point, A, B, C]  
[mratio, X<sub>2</sub>, A, C, r<sub>2</sub>, r<sub>3</sub>]  
[mratio, X<sub>3</sub>, B, A, r<sub>1</sub>, r<sub>2</sub>]  
[foot, L, A, X<sub>2</sub>, X<sub>3</sub>]  
[inter, M, [tline, B, L, X<sub>2</sub>], [tline, L, L, A]]  
[foot, N, C, M, L]



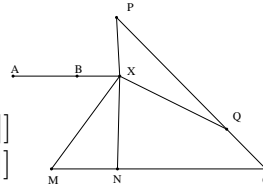
**Example 58** The constraints: [coll, M, A, B], [aratio, D, E, F, D, E, M, 2, 1]. The following points and their construction order are given: A, B, D, E, F.

The construction sequence is  
[point, A, B]  
[point, D]  
[point, E]  
[point, F]  
[mid, X<sub>0</sub>, F, D]  
[inter, M, [line, A, B], [pline, X<sub>0</sub>, D, E]]



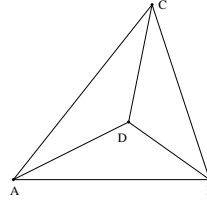
**Example 59** The constraints: [coll, X, A, B], [aratio, X, P, Q, X, M, N, 1, 1]. The following points and their construction order are given: A, B, P, Q, M, N.

The construction sequence is  
[point, A, B]  
[point, P, Q, M, N]  
[inter, X<sub>0</sub>, [line, M, N], [line, P, Q]]  
[inter, X<sub>1</sub>, [tline, X<sub>0</sub>, X<sub>0</sub>, P], [rcir, X<sub>0</sub>, [dis, m, n]]]  
[inter, X<sub>2</sub>, [tline, X<sub>0</sub>, X<sub>0</sub>, M], [rcir, X<sub>0</sub>, [dis, p, q]]]  
[inter, X<sub>3</sub>, [pline, X<sub>1</sub>, X<sub>0</sub>, P], [pline, X<sub>2</sub>, X<sub>0</sub>, M]]  
[inter, X, [line, A, B], [line, X<sub>0</sub>, X<sub>3</sub>]]



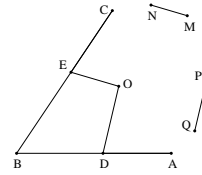
**Example 60** The constraints: [aratio, A, B, D, A, C, D, 1, 1], [aratio, A, B, D, B, C, D, 1, 1]. The following points and their construction order are given: A, B, C.

The construction sequence is  
 [point, A, B, C]  
 [mid, X<sub>0</sub>, B, C]  
 [mid, X<sub>1</sub>, A, C]  
 [inter, D, [line, A, X<sub>0</sub>], [line, B, X<sub>1</sub>]]



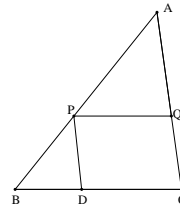
**Example 61** The constraints: [para, O, E, M, N], [sratio, O, E, O, D, r1, r2], [coll, E, A, B], [coll, D, B, C], [para, O, D, P, Q]. The following points and their construction order are given: A, B, C, M, N, P, Q.

The construction sequence is  
 [point, A, B, C]  
 [point, M, N, P, Q]  
 [inter, X<sub>0</sub>, [pline, B, M, N], [rcir, B, r1]]  
 [inter, X<sub>1</sub>, [pline, B, P, Q], [rcir, B, r2]]  
 [inter, X<sub>2</sub>, [pline, X<sub>0</sub>, B, A], [pline, X<sub>1</sub>, B, C]]  
 [on, O, [line, B, X<sub>2</sub>]]  
 [inter, E, [line, A, B], [pline, O, M, N]]  
 [inter, D, [line, B, C], [pline, O, P, Q]]



**Example 62** The constraints: [coll, D, B, C], [coll, P, A, B], [coll, Q, A, C], [para, P, Q, B, C], [sratio, P, D, P, Q, r1, r2]. The following points and their construction order are given: A, B, C, D.

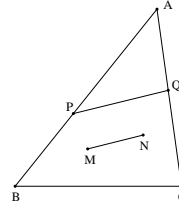
The construction sequence is  
 [point, A, B, C]  
 [on, D, [line, B, C]]  
 [lratio, X<sub>0</sub>, B, C, r1, r2]  
 [inter, X<sub>1</sub>, [line, A, D], [pcir, B, X<sub>0</sub>]]  
 [inter, P, [line, A, B], [pline, D, B, X<sub>1</sub>]]  
 [inter, Q, [line, A, C], [pline, P, B, C]]



**Example 63** The constraints: [coll, P, A, B], [coll, Q, A, C], [para, P, Q, M, N], [cong, A, P, C, Q]. The following points and their construction order are given: A, B, C, M, N.

The construction sequence is

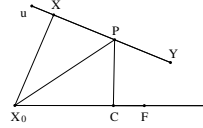
[point,  $A, B, C, M, N$ ]  
 [inter,  $X_0$ , [bi\_line,  $C, A, B$ ], [pline,  $C, M, N$ ]]  
 [inter,  $P$ , [line,  $A, B$ ], [pline,  $X_0, A, C$ ]]  
 [inter,  $Q$ , [line,  $A, C$ ], [pline,  $P, M, N$ ]]



**Example 64** The constraints: [coll,  $P, X, Y$ ], [coll,  $C, E, F$ ], [perp,  $P, C, E, F$ ], [cong,  $P, X, P, C$ ]. The following points and their construction order are given:  $X, Y, E, F$ .

The construction sequence is

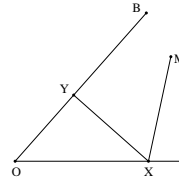
[point,  $X, Y, E, F$ ]  
 [inter,  $X_0$ , [line,  $E, F$ ], [tline,  $X, X, Y$ ]]  
 [inter,  $P$ , [bi\_line,  $X_0, X, E$ ], [line,  $X, Y$ ]]  
 [foot,  $C, P, E, F$ ]



**Example 65** The constraints: [cong,  $X, M, X, Y$ ], [perp,  $X, Y, O, B$ ], [coll,  $O, A, X$ ], [coll,  $O, B, Y$ ]. The following points and their construction order are given:  $O, A, B, M$ .

The construction sequence is

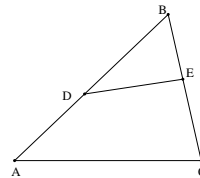
[point,  $O, A, B, M$ ]  
 [foot,  $X_0, A, B, O$ ]  
 [inter,  $X_1$ , [line,  $O, M$ ], [pcir,  $A, X_0$ ]]  
 [inter,  $X$ , [line,  $O, A$ ], [pline,  $M, A, X_1$ ]]  
 [foot,  $Y, X, O, B$ ]



**Example 66** The constraints: [coll,  $D, A, B$ ], [coll,  $E, B, C$ ], [cong,  $A, D, D, E$ ], [cong,  $D, E, E, C$ ]. The following points and their construction order are given:  $A, B, C$ .

The construction sequence is

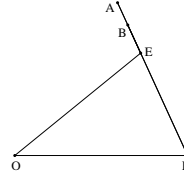
[point,  $A, B, C$ ]  
 [inter,  $X_0$ , [line,  $A, B$ ], [dcir,  $A, C, B$ ]]  
 [inter,  $X_1$ , [pline,  $X_0, A, C$ ], [pcir,  $B, C$ ]]  
 [inter,  $D$ , [line,  $A, B$ ], [line,  $C, X_1$ ]]  
 [inter,  $E$ , [bline,  $D, C$ ], [line,  $B, C$ ]]



**Example 67** The constraints: [angle,  $E, O, F, a1$ ], [dis,  $E, F, d2$ ], [coll,  $E, A, B$ ], [coll,  $F, A, B$ ]. The following points and their construction order are given:  $O, A, B$ .

The construction sequence is

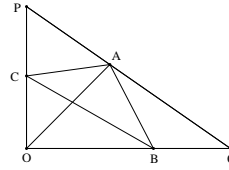
```
[point, O, A, B]
[inter, X0, [line, A, B], [rcir, A, d2]]
[inter, X1, [pline, O, A, B], [a_cir, A, X0, a1]]
[inter, E, [line, A, B], [pline, O, A, X1]]
[inter, F, [line, A, B], [pline, O, X0, X1]]
```



**Example 68** The constraints: [perp, O, P, O, Q], [coll, A, P, Q], [coll, B, O, Q], [coll, C, O, P], [dis, O, P, d1], [eqangle, B, O, A, A, O, C]. The following points and their construction order are given: A, B, C.

The construction sequence is

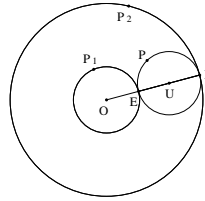
```
[point, A, B, C]
[inter, O, [a_cir, A, C, [d, pi2, 2]], [di_cir, C, B]]
[inter, P, [line, C, O], [rcir, O, d1]]
[inter, Q, [line, A, P], [line, B, O]]
```



**Example 69** The constraints: [coll, U, E, F], [cong, O, E, O, P1], [cong, O, F, O, P2], [coll, O, E, F], [cong, U, E, U, F], [cong, U, F, U, P]. The following points and their construction order are given: O, P1, P2, P.

The construction sequence is

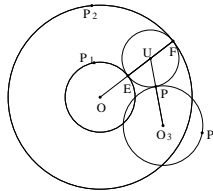
```
[point, O, P1, P2, P]
[inter, X0, [line, O, P1], [pcir, O, P2]]
[mid, X4, P1, X0]
[inter, U, [pcir, O, X4], [dcir, P, P1, X4]]
[inter, E, [pcir, O, P1], [pcir, U, P]]
[inter, F, [pcir, O, P2], [pcir, U, P]]
```



**Example 70** The constraints: [coll, U, E, F], [cong, U, E, U, F], [cong, U, F, U, P], [cong, O, E, O, P1], [cong, O, F, O, P2], [coll, O, E, F], [cong, O3, P3, O3, P], [coll, O3, P, U]. The following points and their construction order are given: O, P1, P2, O3, P3.

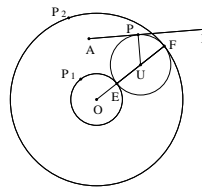
The construction sequence is

```
[point, O, P1, P2, O3, P3]
[inter, X0, [line, O, P1], [pcir, O, P2]]
[mid, X5, P1, X0]
[inter, X7, [line, O3, P3], [dcir, P3, P1, X5]]
[inter, U, [pcir, O, X5], [pcir, O3, X7]]
[inter, P, [line, O3, U], [pcir, O3, P3]]
[inter, E, [pcir, O, P1], [pcir, U, P]]
[inter, F, [pcir, O, P2], [pcir, U, P]]
```





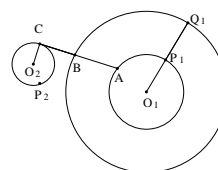
**Example 71** The constraints:  $[\text{coll}, U, E, F]$ ,  $[\text{cong}, O, E, O, P_1]$ ,  $[\text{cong}, O, F, O, P_2]$ ,  $[\text{coll}, O, E, F]$ ,  $[\text{cong}, U, E, U, F]$ ,  $[\text{coll}, P, A, B]$ ,  $[\text{cong}, U, P, U, E]$ ,  $[\text{perp}, U, P, A, B]$ . The following points and their construction order are given:  $O, P_1, P_2, A, B$ .



The construction sequence is

```
[point, O, P1, P2, A, B]
[inter, X0, [line, O, P1], [pcir, O, P2]]
[mid, X6, P1, X0]
[inter, U, [d_line, A, B, [dis, p1, x6]], [pcir, O, X6]]
[inter, E, [dcir, U, P1, X6], [pcir, O, P1]]
[inter, F, [pcir, O, P2], [pcir, U, E]]
[foot, P, U, A, B]
```

**Example 72** The constraints:  $[\text{cong}, O_1, P_1, O_1, A]$ ,  $[\text{cong}, O_1, Q_1, O_1, B]$ ,  $[\text{dis}, A, B, d1]$ ,  $[\text{cong}, O_2, P_2, O_2, C]$ ,  $[\text{coll}, A, B, C]$ ,  $[\text{perp}, O_2, C, C, A]$ . The following points and their construction order are given:  $O_1, P_1, Q_1, O_2, P_2$ .



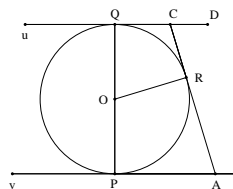
The construction sequence is

```
[point, O1, P1, Q1]
[point, O2, P2]
[inter, X0, [pcir, O1, Q1], [rcir, P1, d1]]
[foot, X1, O1, P1, X0]
[inter, A, [taa_line, O1, X1, O2, P2], [pcir, O1, P1]]
[inter, C, [taa_line, O1, X1, O2, P2], [pcir, O2, P2]]
[inter, B, [line, A, C], [pcir, O1, Q1]]
```

**Example 73** The constraints:  $[\text{para}, A, B, C, D]$ ,  $[\text{cong}, O, P, O, Q]$ ,  $[\text{cong}, O, R, O, P]$ ,  $[\text{coll}, P, A, B]$ ,  $[\text{coll}, Q, C, D]$ ,  $[\text{coll}, R, A, C]$ ,  $[\text{perp}, O, P, A, B]$ ,  $[\text{perp}, O, Q, C, D]$ ,  $[\text{perp}, O, R, A, C]$ . The following points and their construction order are given:  $A, B, C, D$ .

The construction sequence is

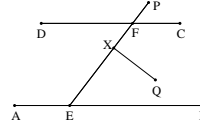
```
[point, A, B, C]
[on, D, [pline, C, A, B]]
[inter, O, [bi_line, C, A, D], [m_line, C, A, B]]
[foot, P, O, A, B]
[foot, Q, O, C, D]
[foot, R, O, A, C]
```



**Example 74** The constraints:  $[\text{para}, A, B, C, D]$ ,  $[\text{coll}, E, A, B]$ ,  $[\text{coll}, F, C, D]$ ,  $[\text{coll}, P, E, F]$ ,  $[\text{coll}, X, E, F]$ ,  $[\text{sratio}, E, X, X, F, r1, r2]$ ,  $[\text{perp}, X, P, X, Q]$ . The following points and their construction order are given:  $A, B, C, D, P, Q$ .

The construction sequence is

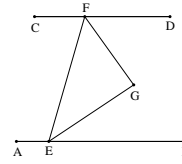
[point,  $A, B, C$ ]  
 [on,  $D$ , [pline,  $C, A, B$ ]]  
 [point,  $P, Q$ ]  
 [mratio,  $X_0, A, C, r_1, r_2$ ]  
 [inter,  $X$ , [pline,  $X_0, A, B$ ], [di\_cir,  $P, Q$ ]]  
 [inter,  $E$ , [line,  $A, B$ ], [line,  $X, P$ ]]  
 [inter,  $F$ , [line,  $C, D$ ], [line,  $P, E$ ]]



**Example 75** The constraints: [para,  $A, B, C, D$ ], [angle,  $E, G, F, a_1$ ], [dis,  $E, F, d_1$ ], [coll,  $E, A, B$ ], [coll,  $F, C, D$ ]. The following points and their construction order are given:  $A, B, C, D, G$ .

The construction sequence is

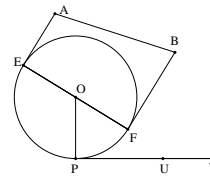
[point,  $A, B, C$ ]  
 [on,  $D$ , [pline,  $C, A, B$ ]]  
 [point,  $G$ ]  
 [inter,  $X_0$ , [line,  $C, D$ ], [rcir,  $A, d_1$ ]]  
 [inter,  $X_1$ , [pline,  $G, C, D$ ], [a\_cir,  $A, X_0, a_1$ ]]  
 [inter,  $E$ , [line,  $A, B$ ], [pline,  $G, A, X_1$ ]]  
 [inter,  $F$ , [line,  $C, D$ ], [pline,  $G, X_0, X_1$ ]]



**Example 76** The constraints: [dis,  $O, E, d_1$ ], [dis,  $F, O, d_1$ ], [dis,  $P, O, d_1$ ], [para,  $A, E, B, F$ ], [perp,  $E, A, E, O$ ], [coll,  $E, O, F$ ], [perp,  $P, O, U, V$ ], [coll,  $P, U, V$ ]. The following points and their construction order are given:  $U, V, A, B$ .

The construction sequence is

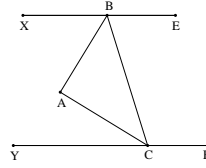
[point,  $U, V, A, B$ ]  
 [mid,  $X_2, B, A$ ]  
 [inter,  $X_3$ , [di\_cir,  $X_2, B$ ], [rcir,  $B, d_1$ ]]  
 [inter,  $O$ , [d\_line,  $U, V, d_1$ ], [line,  $X_2, X_3$ ]]  
 [inter,  $E$ , [di\_cir,  $A, O$ ], [rcir,  $O, d_1$ ]]  
 [inter,  $F$ , [line,  $E, O$ ], [pline,  $B, A, E$ ]]  
 [foot,  $P, O, U, V$ ]



**Example 77** The constraints: [para,  $X, E, Y, F$ ], [coll,  $B, X, E$ ], [coll,  $C, Y, F$ ], [dis,  $B, C, d_1$ ], [perp,  $A, B, A, C$ ]. The following points and their construction order are given:  $A, X, E, Y, F$ .

The construction sequence is

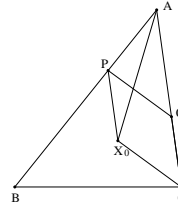
[point, A, X, E, Y]  
 [on, F, [pline, Y, X, E]]  
 [inter, X<sub>0</sub>, [line, F, Y], [rcir, E, d1]]  
 [inter, X<sub>1</sub>, [pline, A, E, X], [di\_cir, E, X<sub>0</sub>]]  
 [inter, B, [line, X, E], [pline, A, E, X<sub>1</sub>]]  
 [inter, C, [line, Y, F], [tline, A, A, B]]



**Example 78** The constraints: [coll, P, A, B], [coll, Q, A, C], [dis, P, Q, d1], [cong, A, P, C, Q]. The following points and their construction order are given: A, B, C.

The construction sequence is

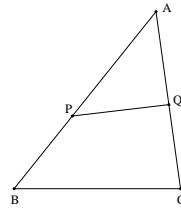
[point, A, B, C]  
 [inter, X<sub>0</sub>, [bi\_line, C, A, B], [rcir, C, d1]]  
 [inter, P, [line, A, B], [pline, X<sub>0</sub>, A, C]]  
 [inter, Q, [line, A, C], [pline, P, C, X<sub>0</sub>]]



**Example 79** The constraints: [coll, P, A, B], [coll, Q, A, C], [cong, A, Q, Q, P], [cong, A, P, C, Q]. The following points and their construction order are given: A, B, C.

The construction sequence is

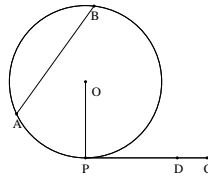
[point, A, B, C]  
 [inter, X<sub>0</sub>, [line, C, B], [pcir, C, A]]  
 [inter, X<sub>2</sub>, [bi\_line, C, A, B], [line, C, X<sub>0</sub>]]  
 [inter, P, [line, A, B], [pline, X<sub>2</sub>, A, C]]  
 [inter, Q, [line, A, C], [pline, P, C, X<sub>0</sub>]]



**Example 80** The constraints: [cong, O, P, O, A], [cong, O, A, O, B], [coll, P, C, D], [perp, O, P, C, D]. The following points and their construction order are given: A, B, C, D.

The construction sequence is

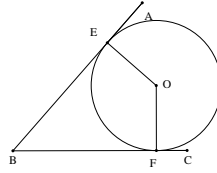
[point, A, B, C, D]  
 [inter, X<sub>0</sub>, [bline, A, B], [line, C, D]]  
 [on, X<sub>1</sub>, [bline, A, B]]  
 [foot, X<sub>2</sub>, X<sub>1</sub>, C, D]  
 [inter, X<sub>3</sub>, [line, X<sub>0</sub>, A], [pcir, X<sub>1</sub>, X<sub>2</sub>]]  
 [inter, O, [bline, A, B], [pline, A, X<sub>3</sub>, X<sub>1</sub>]]  
 [foot, P, O, C, D]



**Example 81** The constraints: [coll, B, C, F], [dis, B, F, d1], [coll, B, A, E], [perp, O, F, B, C], [perp, O, E, A, B], [cong, O, E, O, F]. The following points and their construction order are given: A, B, C.

The construction sequence is

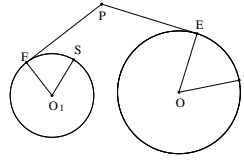
```
[point, A, B, C]
[inter, F, [line, B, C], [rcir, B, d1]]
[inter, E, [line, B, A], [pcir, B, F]]
[inter, O, [tline, E, A, B], [tline, F, B, C]]
```



**Example 82** The constraints: [cong,  $O, E, O, T$ ], [perp,  $O, E, E, P$ ], [cong,  $O_1, F, O_1, S$ ], [perp,  $O_1, F, F, P$ ], [dis,  $E, P, a1$ ], [dis,  $F, P, a2$ ]. The following points and their construction order are given:  $O, T, O_1, S$ .

The construction sequence is

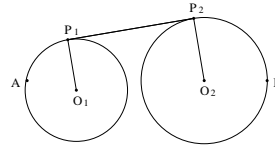
```
[point, O, T, O_1, S]
[inter, X_0, [tline, T, T, O], [rcir, T, a1]]
[inter, X_1, [tline, S, S, O_1], [rcir, S, a2]]
[inter, P, [pcir, O, X_0], [pcir, O_1, X_1]]
[inter, E, [di_cir, O, P], [pcir, O, T]]
[inter, F, [di_cir, O_1, P], [pcir, O_1, S]]
```



**Example 83** The constraints: [cong,  $O_1, P_1, O_1, A$ ], [cong,  $O_2, P_2, O_2, B$ ], [perp,  $O_1, A, A, B$ ], [perp,  $O_2, B, B, A$ ]. The following points and their construction order are given:  $O_1, P_1, O_2, P_2$ .

The construction sequence is

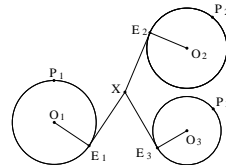
```
[point, O_1, P_1, O_2, P_2]
[inter, X_0, [line, O_1, P_1], [rcir, P_1, [dis, o2, p2]]]
[inter, X_1, [di_cir, O_1, O_2], [pcir, O_1, X_0]]
[inter, A, [line, O_1, X_1], [pcir, O_1, P_1]]
[inter, B, [tline, A, O_1, A], [pcir, O_2, P_2]]
```



**Example 84** The constraints: [cong,  $E_1, O_1, O_1, P_1$ ], [perp,  $O_1, E_1, E_1, X$ ], [cong,  $E_2, O_2, O_2, P_2$ ], [perp,  $O_2, E_2, E_2, X$ ], [cong,  $E_3, O_3, O_3, P_3$ ], [perp,  $O_3, E_3, E_3, X$ ], [cong,  $X, E_1, X, E_2$ ], [cong,  $X, E_1, X, E_3$ ]. The following points and their construction order are given:  $O_1, P_1, O_2, P_2, O_3, P_3$ .

The construction sequence is

```
[point, O_1, P_1, O_2, P_2, O_3, P_3]
[on, X_0, [tline, P_1, P_1, O_1]]
[inter, X_1, [tline, P_2, P_2, O_2], [dcir, P_2, P_1, X_0]]
[inter, X_2, [pcir, O_1, X_0], [pcir, O_2, X_1]]
[on, X_3, [tline, P_1, P_1, O_1]]
[inter, X_4, [tline, P_2, P_2, O_2], [dcir, P_2, P_1, X_3]]
```



```

[inter, X5, [pcir, O1, X3], [pcir, O2, X4]]
[on, X6, [tline, P1, P1, O1]]
[inter, X7, [tline, P3, P3, O3], [dcir, P3, P1, X6]]
[inter, X8, [pcir, O1, X6], [pcir, O3, X7]]
[on, X9, [tline, P1, P1, O1]]
[inter, X10, [tline, P3, P3, O3], [dcir, P3, P1, X9]]
[inter, X11, [pcir, O1, X9], [pcir, O3, X10]]
[inter, X, [line, X2, X5], [line, X8, X11]]
[inter, E1, [di_cir, O1, X], [pcir, O1, P1]]
[inter, E2, [di_cir, O2, X], [pcir, O2, P2]]
[inter, E3, [di_cir, O3, X], [pcir, O3, P3]]

```

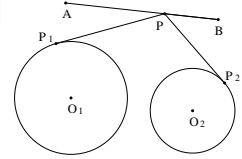
**Example 85** The constraints:  $[coll, P, A, B]$ ,  $[dis, O_1, P_1, d1]$ ,  $[dis, O_2, P_2, d2]$ ,  $[perp, O_1, P_1, P_1, P]$ ,  $[perp, O_2, P_2, P_2, P]$ ,  $[sq\_sum, P_1, P, P_2, s1]$ . The following points and their construction order are given:  $A, B, O_1, O_2$ .

The construction sequence is

```

[point, A, B, O1, O2]
[inter, X0, [line, O1, O2], [rcir, O1, [sum, s1, [m, d1, d1], [m, d2, d2]]]]
[on, X1, [di_cir, O1, X0]]
[inter, X2, [dcir, O1, O1, X1], [dcir, O2, X1, X0]]
[mid, X3, O1, O2]
[inter, P, [line, A, B], [pcir, X3, X2]]
[inter, P1, [di_cir, O1, P], [rcir, O1, d1]]
[inter, P2, [di_cir, O2, P], [rcir, O2, d2]]

```



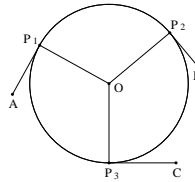
**Example 86** The constraints:  $[cong, O, P_1, O, P_2]$ ,  $[cong, O, P_1, O, P_3]$ ,  $[perp, O, P_1, P_1, A]$ ,  $[perp, O, P_2, P_2, B]$ ,  $[perp, O, P_3, P_3, C]$ ,  $[dis, P_1, A, d1]$ ,  $[dis, P_2, B, d2]$ ,  $[dis, P_3, C, d3]$ . The following points and their construction order are given:  $A, B, C$ .

The construction sequence is

```

[point, A, B, C]
[on, X0, [rcir, A, d1]]
[on, X1, [rcir, B, d2]]
[on, X2, [tline, X0, X0, A]]
[inter, X3, [tline, X1, X1, B], [dcir, X1, X0, X2]]
[inter, X4, [pcir, A, X2], [pcir, B, X3]]
[on, X5, [tline, X0, X0, A]]
[inter, X6, [tline, X1, X1, B], [dcir, X1, X0, X5]]
[inter, X7, [pcir, A, X5], [pcir, B, X6]]
[on, X8, [rcir, A, d1]]
[on, X9, [rcir, C, d3]]
[on, X10, [tline, X8, X8, A]]

```

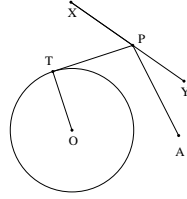


[inter,  $X_{11}$ , [tline,  $X_9, X_9, C$ ], [dcir,  $X_9, X_8, X_{10}$ ]]  
 [inter,  $X_{12}$ , [pcir,  $A, X_{10}$ ], [pcir,  $C, X_{11}$ ]]  
 [on,  $X_{13}$ , [tline,  $X_8, X_8, A$ ]]  
 [inter,  $X_{14}$ , [tline,  $X_9, X_9, C$ ], [dcir,  $X_9, X_8, X_{13}$ ]]  
 [inter,  $X_{15}$ , [pcir,  $A, X_{13}$ ], [pcir,  $C, X_{14}$ ]]  
 [inter,  $O$ , [line,  $X_{12}, X_{15}$ ], [line,  $X_4, X_7$ ]]  
 [inter,  $P_1$ , [di\_cir,  $O, A$ ], [rcir,  $A, d1$ ]]  
 [inter,  $P_3$ , [di\_cir,  $O, C$ ], [rcir,  $C, d3$ ]]  
 [inter,  $P_2$ , [di\_cir,  $O, B$ ], [rcir,  $B, d2$ ]]

**Example 87** The constraints: [coll,  $P, X, Y$ ], [cong,  $P, A, P, T$ ], [dis,  $O, T, d1$ ], [perp,  $O, T, T, P$ ]. The following points and their construction order are given:  $O, X, Y, A$ .

The construction sequence is

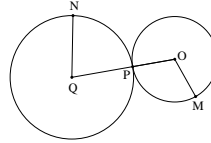
[point,  $O, X, Y, A$ ]  
 [inter,  $X_0$ , [line,  $O, A$ ], [rcir,  $O, d1$ ]]  
 [on,  $X_1$ , [tline,  $X_0, O, A$ ]]  
 [inter,  $X_2$ , [dcir,  $A, X_1, X_0$ ], [dcir,  $O, O, X_1$ ]]  
 [inter,  $P$ , [line,  $X, Y$ ], [tline,  $X_2, O, A$ ]]  
 [inter,  $T$ , [pcir,  $P, A$ ], [rcir,  $O, d1$ ]]



**Example 88** The constraints: [cong,  $O, M, O, P$ ], [dis,  $O, M, r$ ], [cong,  $Q, N, Q, P$ ], [coll,  $O, P, Q$ ]. The following points and their construction order are given:  $Q, M, N$ .

The construction sequence is

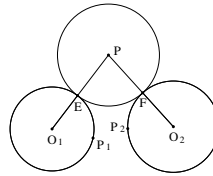
[point,  $Q, M, N$ ]  
 [inter,  $X_1$ , [line,  $Q, N$ ], [rcir,  $N, r$ ]]  
 [inter,  $O$ , [pcir,  $Q, X_1$ ], [rcir,  $M, r$ ]]  
 [inter,  $P$ , [pcir,  $O, M$ ], [pcir,  $Q, N$ ]]



**Example 89** The constraints: [cong,  $O_1, E, O_1, P_1$ ], [cong,  $O_2, F, O_2, P_2$ ], [coll,  $P, E, O_1$ ], [coll,  $P, F, O_2$ ], [dis,  $P, E, r$ ], [cong,  $P, E, P, F$ ]. The following points and their construction order are given:  $O_1, P_1, O_2, P_2$ .

The construction sequence is

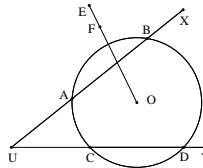
[point,  $O_1, P_1, O_2, P_2$ ]  
 [inter,  $X_4$ , [line,  $O_1, P_1$ ], [rcir,  $P_1, r$ ]]  
 [inter,  $X_5$ , [line,  $O_2, P_2$ ], [rcir,  $P_2, r$ ]]  
 [inter,  $P$ , [pcir,  $O_1, X_4$ ], [pcir,  $O_2, X_5$ ]]  
 [inter,  $E$ , [pcir,  $O_1, P_1$ ], [rcir,  $P, r$ ]]  
 [inter,  $F$ , [pcir,  $O_2, P_2$ ], [pcir,  $P, E$ ]]



**Example 90** The constraints:  $[\text{dis}, U, A, d1]$ ,  $[\text{cong}, U, A, U, B]$ ,  $[\text{cong}, U, A, U, C]$ ,  $[\text{cong}, U, A, U, D]$ ,  $[\text{cong}, A, B, C, D]$ ,  $[\text{coll}, O, X, A]$ ,  $[\text{coll}, O, X, B]$ ,  $[\text{coll}, O, Y, C]$ ,  $[\text{coll}, O, Y, D]$ ,  $[\text{coll}, U, E, F]$ . The following points and their construction order are given:  $O, X, Y, E, F$ .

The construction sequence is

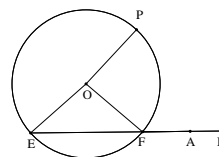
[point,  $O, X, Y, E, F$ ]  
 [inter,  $U$ , [bi\_line,  $O, X, Y$ ], [line,  $E, F$ ]]  
 [inter,  $A$ , [line,  $O, X$ ], [rcir,  $U, d1$ ]]  
 [inter,  $B$ , [line,  $A, O$ ], [pcir,  $U, A$ ]]  
 [inter,  $C$ , [line,  $O, Y$ ], [pcir,  $U, A$ ]]  
 [inter,  $D$ , [line,  $C, O$ ], [pcir,  $U, C$ ]]



**Example 91** The constraints:  $[\text{dis}, O, E, d1]$ ,  $[\text{dis}, O, F, d1]$ ,  $[\text{dis}, O, P, d1]$ ,  $[\text{dis}, E, F, d2]$ ,  $[\text{coll}, E, A, B]$ ,  $[\text{coll}, F, A, B]$ . The following points and their construction order are given:  $A, B, P$ .

The construction sequence is

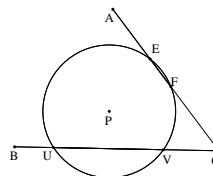
[point,  $A, B, P$ ]  
 [inter,  $X_4$ , [line,  $A, B$ ], [rcir,  $A, d2$ ]]  
 [inter,  $X_5$ , [rcir,  $A, d1$ ], [rcir,  $X_4, d1$ ]]  
 [inter,  $O$ , [pline,  $X_5, A, B$ ], [rcir,  $P, d1$ ]]  
 [inter,  $E$ , [line,  $A, B$ ], [pline,  $O, X_5, X_4$ ]]  
 [inter,  $F$ , [line,  $A, B$ ], [pline,  $O, X_5, A$ ]]



**Example 92** The constraints:  $[\text{dis}, P, E, d1]$ ,  $[\text{dis}, F, P, d1]$ ,  $[\text{dis}, P, U, d1]$ ,  $[\text{dis}, P, V, d1]$ ,  $[\text{dis}, E, F, d2]$ ,  $[\text{dis}, U, V, d3]$ ,  $[\text{coll}, E, O, A]$ ,  $[\text{coll}, F, O, A]$ ,  $[\text{coll}, U, O, B]$ ,  $[\text{coll}, V, O, B]$ . The following points and their construction order are given:  $O, A, B$ .

The construction sequence is

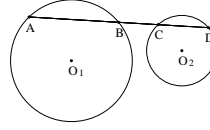
[point,  $O, A, B$ ]  
 [inter,  $X_2$ , [line,  $A, O$ ], [rcir,  $A, d2$ ]]  
 [inter,  $X_3$ , [rcir,  $A, d1$ ], [rcir,  $X_2, d1$ ]]  
 [inter,  $X_6$ , [line,  $B, O$ ], [rcir,  $B, d3$ ]]  
 [inter,  $X_7$ , [rcir,  $B, d1$ ], [rcir,  $X_6, d1$ ]]  
 [inter,  $P$ , [pline,  $X_3, A, O$ ], [pline,  $X_7, B, O$ ]]  
 [inter,  $E$ , [line,  $O, A$ ], [pline,  $P, X_3, X_2$ ]]  
 [inter,  $F$ , [line,  $O, A$ ], [pline,  $P, X_3, A$ ]]  
 [inter,  $U$ , [line,  $O, B$ ], [pline,  $P, X_7, X_6$ ]]  
 [inter,  $V$ , [line,  $O, B$ ], [pline,  $P, X_7, B$ ]]



**Example 93** The constraints:  $[\text{cong}, O_1, P_1, O_1, A]$ ,  $[\text{cong}, O_1, P_1, O_1, B]$ ,  $[\text{cong}, O_2, P_2, O_2, C]$ ,  $[\text{cong}, O_2, P_2, O_2, D]$ ,  $[\text{dis}, A, B, d1]$ ,  $[\text{dis}, C, D, d2]$ ,  $[\text{coll}, A, B, C]$ ,  $[\text{coll}, A, B, D]$ . The following points and their construction order are given:  $O_1, P_1, O_2, P_2$ .

The construction sequence is

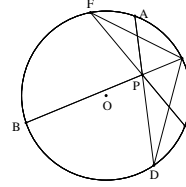
```
[point, O1, P1, O2, P2]
[inter, X0, [pcir, O1, P1], [rcir, P1, d1]]
[mid, X1, P1, X0]
[inter, X2, [pcir, O2, P2], [rcir, P2, d2]]
[mid, X3, P2, X2]
[inter, A, [taa_line, O1, X1, O2, X3], [pcir, O1, P1]]
[inter, C, [taa_line, O1, X1, O2, X3], [pcir, O2, P2]]
[inter, B, [line, A, C], [pcir, O1, A]]
[inter, D, [line, C, A], [pcir, O2, C]]
```



**Example 94** The constraints: [coll, P, A, D], [coll, P, B, E], [coll, P, C, F], [cong, O, A, O, B], [cong, O, A, O, C], [cong, O, A, O, D], [cong, O, A, O, E], [cong, O, A, O, F], [dis, D, E, d1], [dis, E, F, d2]. The following points and their construction order are given: A, B, C, O.

The construction sequence is

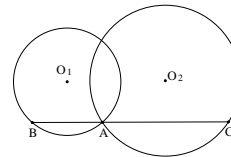
```
[point, A, B, C]
[inter, O, [bline, A, B], [bline, A, C]]
[inter, X0, [pcir, O, A], [rcir, A, d1]]
[inter, X1, [pcir, O, B], [rcir, B, d2]]
[inter, P, [a_cir, A, B, [ang, b, a, x0]], [a_cir, B, C, [ang, c, b, x1]]]
[inter, D, [line, A, P], [pcir, O, A]]
[inter, E, [line, B, P], [pcir, O, B]]
[inter, F, [line, C, P], [pcir, O, C]]
```



**Example 95** The constraints: [coll, A, B, C], [cong, O<sub>1</sub>, A, O<sub>1</sub>, B], [cong, O<sub>2</sub>, A, O<sub>2</sub>, C], [dis, B, C, d1]. The following points and their construction order are given: O<sub>1</sub>, O<sub>2</sub>, A.

The construction sequence is

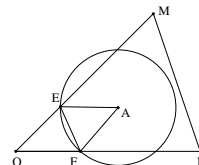
```
[point, O1, O2, A]
[inter, X0, [di_cir, O1, O2], [rcir, O1, [div, d1, 2]]]
[inter, B, [pline, A, O1, X0], [pcir, O1, A]]
[inter, C, [line, A, B], [pcir, O2, A]]
```



**Example 96** The constraints: [cong, A, E, A, F], [coll, O, M, E], [coll, O, N, F], [para, E, F, M, N]. The following points and their construction order are given: O, M, N, A.

The construction sequence is

```
[point, O, M, N, A]
[mid, X0, M, N]
[inter, X1, [line, O, X0], [tline, A, M, N]]
```



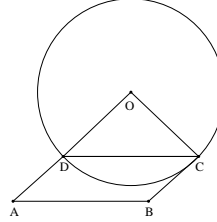


[inter,  $E$ , [line,  $O$ ,  $M$ ], [pline,  $X_1$ ,  $M$ ,  $N$ ]]  
 [inter,  $F$ , [line,  $O$ ,  $N$ ], [pline,  $E$ ,  $M$ ,  $N$ ]]

**Example 97** The constraints: [dis,  $O$ ,  $C$ , d1], [dis,  $O$ ,  $D$ , d1], [para,  $A$ ,  $B$ ,  $C$ ,  $D$ ], [para,  $A$ ,  $D$ ,  $B$ ,  $C$ ]. The following points and their construction order are given:  $O$ ,  $A$ ,  $B$ .

The construction sequence is

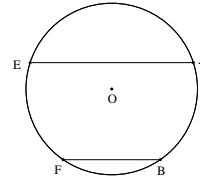
[point,  $O$ ,  $A$ ,  $B$ ]  
 [on,  $X_0$ , [rcir,  $O$ , d1]]  
 [inter,  $X_1$ , [dcir,  $X_0$ ,  $B$ ,  $A$ ], [pcir,  $O$ ,  $X_0$ ]]  
 [mid,  $X_2$ ,  $X_0$ ,  $X_1$ ]  
 [inter,  $X_3$ , [tline,  $O$ ,  $B$ ,  $A$ ], [pcir,  $O$ ,  $X_2$ ]]  
 [inter,  $C$ , [pline,  $X_3$ ,  $B$ ,  $A$ ], [rcir,  $O$ , d1]]  
 [inter,  $D$ , [pline,  $A$ ,  $B$ ,  $C$ ], [pline,  $C$ ,  $A$ ,  $B$ ]]



**Example 98** The constraints: [cong,  $O$ ,  $A$ ,  $O$ ,  $B$ ], [cong,  $O$ ,  $A$ ,  $O$ ,  $E$ ], [cong,  $O$ ,  $A$ ,  $O$ ,  $F$ ], [sratio,  $A$ ,  $E$ ,  $B$ ,  $F$ , r1, r2], [para,  $A$ ,  $E$ ,  $B$ ,  $F$ ]. The following points and their construction order are given:  $O$ ,  $A$ ,  $B$ .

The construction sequence is

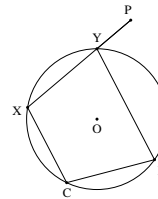
[point,  $O$ ,  $A$ ]  
 [on,  $B$ , [pcir,  $O$ ,  $A$ ]]  
 [mratio,  $X_0$ ,  $A$ ,  $B$ , r1, r2]  
 [inter,  $E$ , [tline,  $A$ ,  $O$ ,  $X_0$ ], [pcir,  $O$ ,  $A$ ]]  
 [inter,  $F$ , [pline,  $B$ ,  $A$ ,  $E$ ], [pcir,  $O$ ,  $B$ ]]



**Example 99** The constraints: [cong,  $O$ ,  $C$ ,  $O$ ,  $B$ ], [cong,  $O$ ,  $B$ ,  $O$ ,  $X$ ], [cong,  $O$ ,  $B$ ,  $O$ ,  $Y$ ], [coll,  $X$ ,  $Y$ ,  $P$ ], [para,  $B$ ,  $Y$ ,  $C$ ,  $X$ ]. The following points and their construction order are given:  $O$ ,  $B$ ,  $C$ ,  $P$ .

The construction sequence is

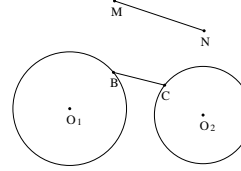
[point,  $O$ ,  $B$ ,  $P$ ]  
 [on,  $C$ , [pcir,  $O$ ,  $B$ ]]  
 [mid,  $X_0$ ,  $C$ ,  $B$ ]  
 [inter,  $X$ , [ta\_line,  $P$ ,  $O$ ,  $X_0$ ], [pcir,  $O$ ,  $B$ ]]  
 [inter,  $Y$ , [line,  $X$ ,  $P$ ], [pcir,  $O$ ,  $X$ ]]



**Example 100** The constraints: [dis,  $O_1$ ,  $B$ , d1], [dis,  $O_2$ ,  $C$ , d2], [para,  $B$ ,  $C$ ,  $M$ ,  $N$ ], [dis,  $B$ ,  $C$ , d3]. The following points and their construction order are given:  $O_1$ ,  $O_2$ ,  $M$ ,  $N$ .

The construction sequence is

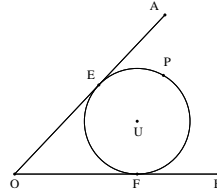
[point,  $O_1, O_2, M, N$ ]  
 [inter,  $X_1, [\text{pline}, O_2, M, N], [\text{rcir}, O_2, d3]$ ]  
 [inter,  $B, [\text{rcir}, O_1, d1], [\text{rcir}, X_1, d2]$ ]  
 [inter,  $C, [\text{pline}, B, M, N], [\text{pline}, O_2, B, X_1]$ ]



**Example 101** The constraints: [cong,  $U, E, U, F$ ], [cong,  $U, E, U, P$ ], [coll,  $E, O, A$ ], [perp,  $U, E, O, A$ ], [coll,  $F, O, B$ ], [perp,  $U, F, O, B$ ]. The following points and their construction order are given:  $O, A, B, P$ .

The construction sequence is

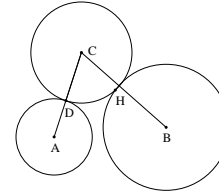
[point,  $O, A, B, P$ ]  
 [inter,  $X_0, [\text{line}, O, P], [\text{pcir}, A, A]$ ]  
 [inter,  $U, [\text{bi\_line}, A, O, B], [\text{pline}, P, A, X_0]$ ]  
 [foot,  $E, U, O, A$ ]  
 [foot,  $F, U, O, B$ ]



**Example 102** The constraints: [coll,  $C, A, D$ ], [coll,  $C, H, B$ ], [dis,  $B, H, d1$ ], [cong,  $C, D, C, H$ ]. The following points and their construction order are given:  $A, B, D$ .

The construction sequence is

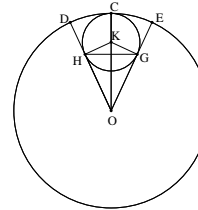
[point,  $A, B, D$ ]  
 [inter,  $X_1, [\text{line}, A, D], [\text{rcir}, D, d1]$ ]  
 [inter,  $C, [\text{bline}, B, X_1], [\text{line}, A, D]$ ]  
 [inter,  $H, [\text{line}, C, B], [\text{rcir}, B, d1], [\text{in\_segment}, B, C]$ ]



**Example 103** The constraints: [cong,  $O, D, O, E$ ], [cong,  $O, C, O, D$ ], [cong,  $K, C, K, H$ ], [cong,  $K, H, K, G$ ], [coll,  $H, O, E$ ], [coll,  $G, O, D$ ], [perp,  $K, H, O, E$ ], [perp,  $K, G, O, D$ ], [coll,  $K, O, C$ ]. The following points and their construction order are given:  $O, D, E$ .

The construction sequence is

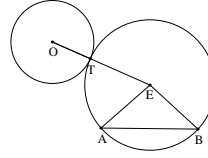
[point,  $O, D$ ]  
 [on,  $E, [\text{pcir}, O, D]$ ]  
 [inter,  $C, [\text{bi\_line}, E, O, D], [\text{pcir}, O, D]$ ]  
 [inter,  $X_0, [\text{line}, E, O], [\text{tline}, C, C, O]$ ]  
 [inter,  $K, [\text{bi\_line}, C, X_0, E], [\text{line}, O, C]$ ]  
 [foot,  $H, K, O, E$ ]  
 [foot,  $G, K, O, D$ ]



**Example 104** The constraints: [cong,  $E, A, E, B$ ], [cong,  $E, A, E, T$ ], [coll,  $O, T, E$ ], [dis,  $O, T, d1$ ]. The following points and their construction order are given:  $A, B, O$ .

The construction sequence is

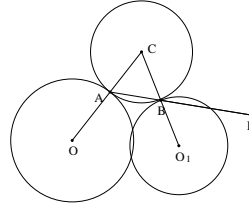
```
[point, A, B, O]
[on, X1, [bline, A, B]]
[inter, X2, [pcir, X1, A], [rcir, O, d1]]
[inter, X3, [pcir, O, X2], [pcir, X1, X2]]
[inter, X4, [line, A, B], [line, X2, X3]]
[inter, T, [di_cir, X4, O], [rcir, O, d1]]
[inter, E, [bline, A, B], [line, O, T]]
```



**Example 105** The constraints: [coll,  $O$ ,  $A$ ,  $C$ ], [coll,  $O_1$ ,  $B$ ,  $C$ ], [cong,  $C$ ,  $A$ ,  $C$ ,  $B$ ], [dis,  $O$ ,  $A$ ,  $d1$ ], [dis,  $O_1$ ,  $B$ ,  $d2$ ], [coll,  $A$ ,  $B$ ,  $P$ ]. The following points and their construction order are given:  $O$ ,  $O_1$ ,  $P$ .

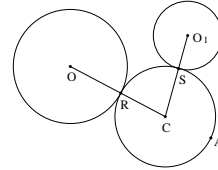
The construction sequence is

```
[point, O, O1, P]
[mratio, X1, O, O1, d1, [r, d2]]
[inter, A, [line, P, X1], [rcir, O, d1]]
[inter, B, [line, A, P], [rcir, O1, d2]]
[inter, C, [line, O, A], [line, O1, B]]
```



**Example 106** The constraints: [cong,  $C$ ,  $R$ ,  $C$ ,  $S$ ], [cong,  $C$ ,  $A$ ,  $C$ ,  $R$ ], [coll,  $C$ ,  $R$ ,  $O$ ], [dis,  $O$ ,  $R$ ,  $d1$ ], [coll,  $C$ ,  $S$ ,  $O_1$ ], [dis,  $O_1$ ,  $S$ ,  $d2$ ]. The following points and their construction order are given:  $O$ ,  $O_1$ ,  $A$ .

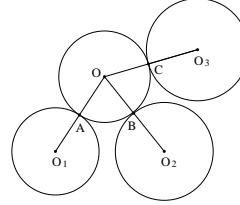
```
[point, O, O1, A]
[mratio, X7, O, O1, d1, [r, d2]]
[inter, X8, [line, A, X7], [rcir, X7, [d, [ess, o, d1, o1, d2], [dis, x7, a]]]]
[on, X9, [bline, A, X8]]
[inter, X10, [pcir, X9, A], [rcir, O, d1]]
[inter, X10, [pcir, X9, A], [rcir, O, d1]]
[inter, X11, [line, A, X8], [line, X10, X10]]
[inter, R, [di_cir, X11, O], [rcir, O, d1]]
[inter, C, [bline, A, R], [line, R, O]]
[inter, S, [line, C, O1], [rcir, O1, d2], [cong, R, C, C, S]]
```



**Example 107** The constraints: [cong,  $A$ ,  $O$ ,  $O$ ,  $B$ ], [cong,  $O$ ,  $A$ ,  $O$ ,  $C$ ], [coll,  $O$ ,  $A$ ,  $O_1$ ], [dis,  $O_1$ ,  $A$ ,  $d1$ ], [coll,  $O$ ,  $B$ ,  $O_2$ ], [dis,  $O_2$ ,  $B$ ,  $d2$ ], [coll,  $O$ ,  $C$ ,  $O_3$ ], [dis,  $O_3$ ,  $C$ ,  $d3$ ]. The following points and their construction order are given:  $O_1$ ,  $O_2$ ,  $O_3$ .

The construction sequence is

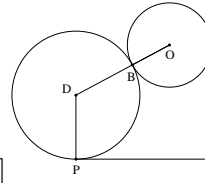
```
[point, O1, O2, O3]
[on, X16, [rcir, O1, d1]]
[on, X17, [rcir, O3, d3]]
[inter, X18, [line, O1, O], [rcir, O, d2]]
[inter, X19, [line, O3, O], [rcir, O, d2]]
[mratio, X25, O1, O3, [dis, o1, x18], [r, [dis, o3, x19]]]
[inter, X26, [line, O2, X25], [rcir, X25, [d, [ess, o1, [dis, o1, x18], o3, [dis, o3, x19]], [dis,
x25, o2]]]]
[on, X27, [bline, O2, X26]]
[inter, X28, [pcir, O1, X18], [pcir, X27, O2]]
[inter, X28, [pcir, O1, X18], [pcir, X27, O2]]
[inter, X29, [line, O2, X26], [line, X28, X28]]
[inter, A, [di_cir, X29, O1], [rcir, O1, d1]]
[inter, X34, [line, A, O1], [rcir, A, d2]]
[inter, O, [bline, O2, X34], [line, A, O1]]
[inter, B, [line, O, O2], [rcir, O2, d2], [cong, A, O, O, B]]
[inter, C, [line, O, O3], [rcir, O3, d3], [cong, A, O, O, C]]
```



**Example 108** The constraints: [perp, D, P, P, Q], [cong, D, P, D, B], [coll, D, B, O], [dis, O, B, d1]. The following points and their construction order are given: P, Q, O.

The construction sequence is

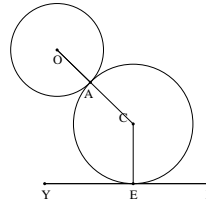
```
[point, P, Q, O]
[inter, X0, [tline, P, P, Q], [rcir, P, d1]]
[inter, D, [bline, O, X0], [tline, P, P, Q]]
[inter, B, [line, D, O], [rcir, O, d1], [cong, P, D, D, B]]
```



**Example 109** The constraints: [perp, C, E, X, Y], [coll, E, X, Y], [cong, C, E, C, A], [coll, C, A, O]. The following points and their construction order are given: X, Y, O, A.

The construction sequence is

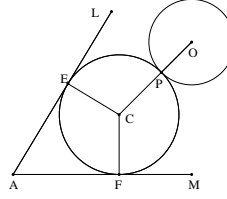
```
[point, X, Y, O, A]
[inter, X0, [line, X, Y], [tline, A, A, O]]
[inter, C, [bi_line, X0, A, X], [line, A, O]]
[foot, E, C, X, Y]
```



**Example 110** The constraints: [perp, C, E, A, L], [coll, E, A, L], [perp, C, F, A, M], [coll, F, A, M], [cong, C, E, C, F], [cong, C, E, C, P], [coll, C, P, O], [dis, P, O, d1]. The following points and their construction order are given: A, L, M, O.

The construction sequence is

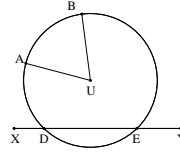
```
[point, A, L, M, O]
[inter, X0, [tline, A, A, L], [rcir, A, d1]]
[inter, X1, [pline, L, A, X0], [pline, X0, A, L]]
[inter, X2, [bi_line, L, A, M], [line, X0, X1]]
[inter, X3, [line, X0, X1], [tline, L, A, L]]
[inter, X4, [line, X2, O], [pcir, L, X3]]
[inter, C, [bi_line, L, A, M], [pline, O, L, X4]]
[foot, E, C, A, L]
[foot, F, C, A, M]
[inter, P, [line, C, O], [rcir, O, d1], [cong, E, C, C, P]]
```



**Example 111** The constraints: [cong,  $D, U, A, U$ ], [cong,  $A, U, B, U$ ], [cong,  $D, U, E, U$ ], [dis,  $D, E, d1$ ], [coll,  $D, X, Y$ ], [coll,  $E, X, Y$ ]. The following points and their construction order are given:  $X, Y, A, B$ .

The construction sequence is

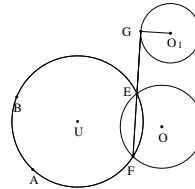
```
[point, X, Y, A, B]
[on, X0, [bline, A, B]]
[inter, X1, [pcir, X0, A], [rcir, A, d1]]
[mid, X2, A, X1]
[inter, X3, [line, A, B], [line, X, Y]]
[inter, X4, [ta_line, X3, X0, X2], [pcir, X0, A]]
[inter, D, [line, X, Y], [rcir, X3, x4]]
[inter, U, [bline, A, B], [bline, D, A]]
[inter, E, [line, X, Y], [rcir, D, d1], [cong, D, U, U, E]]
```



**Example 112** The constraints: [cong,  $E, U, A, U$ ], [cong,  $U, E, U, F$ ], [cong,  $U, A, U, B$ ], [dis,  $O, E, d1$ ], [dis,  $O, F, d1$ ], [coll,  $E, F, G$ ], [perp,  $E, F, G, O_1$ ], [dis,  $O_1, G, d2$ ]. The following points and their construction order are given:  $A, B, O, O_1$ .

The construction sequence is

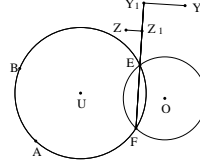
```
[point, A, B, O, O1]
[on, X0, [rcir, O, d1]]
[inter, X1, [c_cir, A, B, X0], [pcir, O, X0]]
[inter, X2, [line, A, B], [line, X0, X1]]
[on, X3, [rcir, O1, d2]]
[inter, E, [ta_line, X2, O1, X3], [rcir, O, d1]]
[inter, U, [bline, A, B], [bline, E, A]]
[inter, G, [di_cir, E, O1], [rcir, O1, d2]]
[inter, F, [line, E, G], [rcir, O, d1], [cong, E, U, U, F], [perp, E, F, G, O1]]
```



**Example 113** The constraints: [cong,  $E, U, A, U$ ], [cong,  $U, E, U, F$ ], [cong,  $U, A, U, B$ ], [dis,  $O, E, d1$ ], [dis,  $O, F, d1$ ], [coll,  $E, F, Y_1$ ], [perp,  $E, F, Y_1, Y$ ], [coll,  $E, F, Z_1$ ], [perp,  $E, F, Z_1, Z$ ], [sratio,  $Y, Y_1, Z, Z_1, r1, r2$ ]. The following points and their construction order are given:  $A, B, O, Y, Z$ .

The construction sequence is

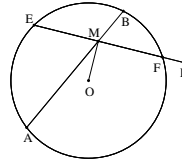
```
[point, A, B, O, Y, Z]
[on, X0, [rcir, O, d1]]
[inter, X1, [c_cir, A, B, X0], [pcir, O, X0]]
[inter, X2, [line, A, B], [line, X0, X1]]
[mratio, X3, Y, Z, r1, r2]
[inter, E, [line, X2, X3], [rcir, O, d1]]
[inter, U, [bline, A, B], [bline, E, A]]
[inter, F, [line, E, X3], [pcir, O, E]]
[mratio, X8, Z, Y, r2, r1]
[foot, Z1, Z, E, F]
[foot, Y1, Y, E, F]
```



**Example 114** The constraints: [cong,  $O, A, O, B$ ], [cong,  $O, A, O, E$ ], [cong,  $O, A, O, F$ ], [coll,  $E, P, F$ ], [coll,  $A, B, M$ ], [mid,  $M, E, F$ ]. The following points and their construction order are given:  $O, A, B, P$ .

The construction sequence is

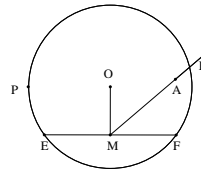
```
[point, O, A]
[on, B, [pcir, O, A]]
[point, P]
[inter, M, [line, A, B], [di_cir, O, P]]
[inter, E, [line, P, M], [pcir, O, A]]
[inter, F, [line, E, M], [pcir, O, E]]
```



**Example 115** The constraints: [coll,  $M, A, B$ ], [cong,  $O, P, O, E$ ], [cong,  $O, P, O, F$ ], [mid,  $M, E, F$ ], [dis,  $E, F, d1$ ]. The following points and their construction order are given:  $O, P, A, B$ .

The construction sequence is

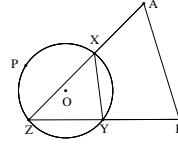
```
[point, O, P, A, B]
[inter, X0, [pcir, O, P], [rcir, P, d1]]
[mid, X1, P, X0]
[inter, M, [line, A, B], [pcir, O, X1]]
[sym, X2, M, O]
[inter, E, [dcir, X2, O, P], [pcir, O, P]]
[sym, F, M, E]
```



**Example 116** The constraints: [mid,  $X, Z, A$ ], [mid,  $Y, Z, B$ ], [cong,  $O, P, O, X$ ], [cong,  $O, P, O, Y$ ]. The following points and their construction order are given:  $O, P, A, B$ .

The construction sequence is

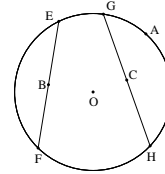
[point,  $O, P, A, B$ ]  
 [sym,  $X_0, O, P$ ]  
 [sym,  $X_1, O, A$ ]  
 [sym,  $X_2, O, P$ ]  
 [sym,  $X_3, O, B$ ]  
 [inter,  $Z, [dcir, X_1, P, X_0], [dcir, X_3, P, X_2]$ ]  
 [mid,  $X, Z, A$ ]  
 [mid,  $Y, Z, B$ ]



**Example 117** The constraints: [cong,  $O, A, O, E$ ], [cong,  $O, A, O, F$ ], [cong,  $O, A, O, G$ ], [cong,  $O, A, O, H$ ], [mid,  $B, E, F$ ], [mid,  $C, G, H$ ], [dis,  $E, F, d1$ ], [dis,  $G, H, d2$ ]. The following points and their construction order are given:  $A, B, C$ .

The construction sequence is

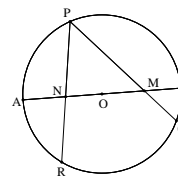
[point,  $A, B, C$ ]  
 [inter,  $X_0, [line, A, C], [rcir, A, [div, [mul, d2, d2], 4]]$ ]  
 [on,  $X_1, [tline, X_0, A, A]$ ]  
 [inter,  $X_2, [dcir, A, A, X_1], [dcir, C, X_1, X_0]$ ]  
 [inter,  $X_3, [line, A, B], [rcir, A, [div, [mul, d1, d1], 4]]$ ]  
 [on,  $X_4, [tline, X_3, A, A]$ ]  
 [inter,  $X_5, [dcir, A, A, X_4], [dcir, B, X_4, X_3]$ ]  
 [inter,  $O, [tline, X_2, A, C], [tline, X_5, A, B]$ ]  
 [sym,  $X_9, B, O$ ]  
 [inter,  $E, [dcir, X_9, O, A], [pcir, O, A]$ ]  
 [sym,  $F, B, E$ ]  
 [sym,  $X_{10}, C, O$ ]  
 [inter,  $G, [dcir, X_{10}, O, A], [pcir, O, A]$ ]  
 [sym,  $H, C, G$ ]



**Example 118** The constraints: [mid,  $O, A, B$ ], [coll,  $A, B, M$ ], [coll,  $A, B, N$ ], [cong,  $P, Q, P, R$ ], [coll,  $P, M, Q$ ], [coll,  $P, N, R$ ], [cong,  $O, A, O, P$ ], [cong,  $O, A, O, Q$ ], [cong,  $O, A, O, R$ ]. The following points and their construction order are given:  $A, B, O, M, N$ .

The construction sequence is

[point,  $A, B$ ]  
 [mid,  $O, A, B$ ]  
 [on,  $M, [line, A, B]$ ]  
 [on,  $N, [line, A, B]$ ]

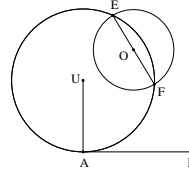


[harmonic,  $X_0, O, M, N$ ]  
 [inter,  $P, [\text{di\_cir}, O, X_0], [\text{pcir}, O, A]$ ]  
 [inter,  $Q, [\text{line}, P, M], [\text{pcir}, O, P]$ ]  
 [inter,  $R, [\text{line}, P, N], [\text{pcir}, O, P]$ ]

**Example 119** The constraints: [cong,  $E, U, A, U$ ], [cong,  $U, E, U, F$ ], [perp,  $U, A, A, B$ ], [mid,  $O, E, F$ ], [dis,  $O, E, d1$ ]. The following points and their construction order are given:  $A, B, O$ .

The construction sequence is

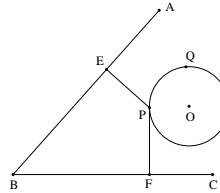
[point,  $A, B, O$ ]  
 [on,  $X_0, [\text{rcir}, O, d1]$ ]  
 [inter,  $X_1, [\text{bline}, X_0, A], [\text{tline}, A, A, B]$ ]  
 [inter,  $X_2, [\text{pcir}, O, X_0], [\text{pcir}, X_1, X_0]$ ]  
 [inter,  $X_3, [\text{line}, A, B], [\text{line}, X_0, X_2]$ ]  
 [inter,  $E, [\text{line}, X_3, O], [\text{rcir}, O, d1]$ ]  
 [inter,  $U, [\text{bline}, E, A], [\text{tline}, A, A, B]$ ]  
 [sym,  $F, O, E$ ]



**Example 120** The constraints: [perp,  $P, E, A, B$ ], [perp,  $P, F, B, C$ ], [sratio,  $P, E, P, F, r1, r2$ ], [coll,  $E, A, B$ ], [coll,  $F, B, C$ ], [cong,  $O, P, O, Q$ ]. The following points and their construction order are given:  $A, B, C, O, Q$ .

The construction sequence is

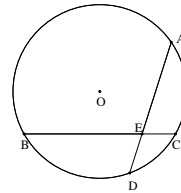
[point,  $A, B, C, O, Q$ ]  
 [inter,  $X_0, [\text{d\_line}, B, A, r1], [\text{d\_line}, B, C, r2]$ ]  
 [inter,  $P, [\text{line}, B, X_0], [\text{pcir}, O, Q]$ ]  
 [foot,  $E, P, A, B$ ]  
 [foot,  $F, P, B, C$ ]



**Example 121** The constraints: [cong,  $O, A, O, B$ ], [cong,  $O, A, O, C$ ], [cong,  $O, A, O, D$ ], [sratio,  $D, A, D, E, r1, r2$ ], [coll,  $A, E, D$ ], [coll,  $E, B, C$ ]. The following points and their construction order are given:  $O, A, B, C$ .

The construction sequence is

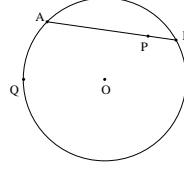
[point,  $O, A$ ]  
 [on,  $B, [\text{pcir}, O, A]$ ]  
 [on,  $C, [\text{pcir}, O, A]$ ]  
 [mratio,  $X_1, A, B, r1, r2$ ]  
 [inter,  $D, [\text{pline}, X_1, B, C], [\text{pcir}, O, A]$ ]  
 [inter,  $E, [\text{line}, A, D], [\text{line}, B, C]$ ]





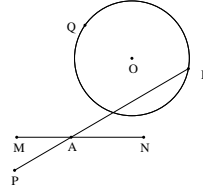
**Example 122** The constraints: [cong,  $O, A, O, Q$ ], [cong,  $O, B, O, Q$ ], [coll,  $P, A, B$ ], [sratio,  $A, P, P, B, r_1, r_2$ ]. The following points and their construction order are given:  $O, P, Q$ .

The construction sequence is  
 [point,  $O, P, Q$ ]  
 [lratio,  $X_0, P, O, r_1, r_2$ ]  
 [lratio,  $X_1, O, Q, r_1, r_2$ ]  
 [inter,  $A, [dcir, X_0, O, X_1], [pcir, O, Q]$ ]  
 [inter,  $B, [line, A, P], [pcir, O, A]$ ]



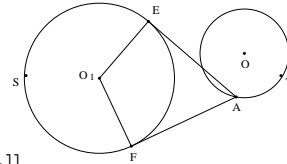
**Example 123** The constraints: [cong,  $O, B, O, Q$ ], [coll,  $P, A, B$ ], [coll,  $A, M, N$ ], [sratio,  $P, A, P, B, r_1, r_2$ ]. The following points and their construction order are given:  $O, P, Q, M, N$ .

The construction sequence is  
 [point,  $O, P, Q, M, N$ ]  
 [lratio,  $X_0, P, M, r_2, r_1$ ]  
 [inter,  $B, [pline, X_0, M, N], [pcir, O, Q]$ ]  
 [inter,  $A, [line, M, N], [line, P, B]$ ]



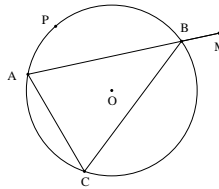
**Example 124** The constraints: [angle,  $E, A, F, m$ ], [cong,  $O, A, O, T$ ], [cong,  $O_1, E, O_1, S$ ], [cong,  $O_1, F, O_1, S$ ], [perp,  $O_1, E, E, A$ ], [perp,  $O_1, F, F, A$ ]. The following points and their construction order are given:  $O, T, O_1, S$ .

The construction sequence is  
 [point,  $O, T, O_1, S$ ]  
 [inter,  $A, [ac\_cir, O_1, S, m], [pcir, O, T]$ ]  
 [inter,  $E, [di\_cir, O_1, A], [pcir, O_1, S]$ ]  
 [inter,  $F, [pcir, O_1, E], [pcir, A, E], [angle, E, A, F, m]$ ]



**Example 125** The constraints: [cong,  $O, A, O, P$ ], [cong,  $O, B, O, P$ ], [cong,  $O, C, O, P$ ], [angle,  $B, A, C, a_1$ ], [angle,  $B, C, A, a_2$ ], [coll,  $A, B, M$ ]. The following points and their construction order are given:  $O, P, M$ .

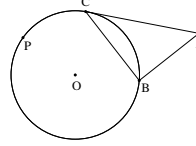
The construction sequence is  
 [point,  $O, P, M$ ]  
 [on,  $X_0, [pcir, O, P]$ ]  
 [inter,  $X_1, [nline, P, X_0, a_2], [pcir, O, P]$ ]  
 [mid,  $X_2, X_0, X_1$ ]  
 [inter,  $A, [ta\_line, M, O, X_2], [pcir, O, P]$ ]  
 [inter,  $B, [line, A, M], [pcir, O, A]$ ]  
 [inter,  $C, [nline, A, B, a_1], [pcir, O, P]$ ]



**Example 126** The constraints: [cong,  $B, O, P, O$ ], [cong,  $C, O, P, O$ ], [dis,  $B, C, d1$ ], [angle,  $B, A, C, a1$ ]. The following points and their construction order are given: $O, P, A$ .

The construction sequence is

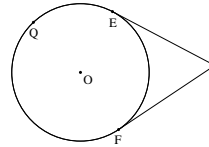
```
[point, O, P, A]
[inter, X0, [pcir, O, P], [rcir, P, d1]]
[on, X1, [nline, P, X0, a1]]
[inter, X2, [bline, P, X0], [tline, P, P, X1]]
[inter, X3, [dcir, A, X2, P], [pcir, O, X2]]
[inter, B, [dcir, X3, P, X2], [pcir, O, P]]
[inter, C, [pcir, O, P], [rcir, B, d1]]
```



**Example 127** The constraints: [dis,  $O, Q, d1$ ], [dis,  $O, E, d1$ ], [dis,  $O, F, d1$ ], [perp,  $O, E, E, P$ ], [perp,  $O, F, F, P$ ], [angle,  $E, P, F, a1$ ]. The following points and their construction order are given: $Q, P$ .

The construction sequence is

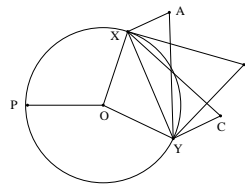
```
[point, Q, P]
[on, X3, [nline, P, Q, [r, a1]]]
[inter, X4, [d_line, P, Q, d1], [d_line, P, X3, d1]]
[inter, O, [pcir, P, X4], [rcir, Q, d1]]
[inter, E, [di_cir, O, P], [rcir, O, d1]]
[inter, F, [pcir, O, E], [pcir, P, E]]
```



**Example 128** The constraints: [cong,  $X, O, O, P$ ], [cong,  $Y, O, O, P$ ], [eqangle,  $X, A, Y, X, B, Y$ ], [eqangle,  $X, A, Y, X, C, Y$ ]. The following points and their construction order are given: $A, B, C, O, P$ .

The construction sequence is

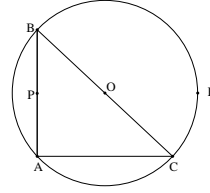
```
[point, A, B, C]
[point, O, P]
[inter, X, [c_cir, A, C, B], [pcir, O, P]]
[inter, Y, [c_cir, A, B, X], [pcir, O, P]]
```



**Example 129** The constraints: [cong,  $O, A, O, E$ ], [cong,  $O, B, O, E$ ], [cong,  $O, C, O, E$ ], [perp,  $A, B, A, C$ ], [angle,  $A, C, B, a1$ ], [coll,  $A, B, P$ ]. The following points and their construction order are given: $O, E, P$ .

The construction sequence is

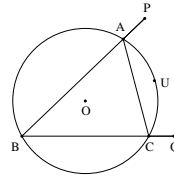
```
[point, O, E, P]
[on, X0, [pcir, O, E]]
[inter, X1, [nline, E, X0, a1], [pcir, O, E]]
[mid, X2, X0, X1]
[inter, A, [ta_line, P, O, X2], [pcir, O, E]]
[inter, B, [line, A, P], [pcir, O, A]]
[inter, C, [tline, A, A, B], [pcir, O, A]]
```



**Example 130** The constraints:  $[\text{cong}, O, U, O, A]$ ,  $[\text{cong}, O, U, O, B]$ ,  $[\text{cong}, O, U, O, C]$ ,  $[\text{angle}, B, A, C, a1]$ ,  $[\text{coll}, P, A, B]$ ,  $[\text{coll}, Q, B, C]$ . The following points and their construction order are given:  $O, U, P, Q$ .

The construction sequence is

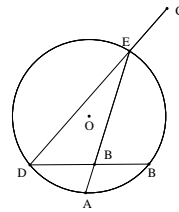
```
[point, O, U, P, Q]
[on, X0, [pcir, O, U]]
[inter, X1, [nline, U, X0, a1], [pcir, O, U]]
[mid, X2, X0, X1]
[inter, B, [ta_line, Q, O, X2], [pcir, O, U]]
[inter, A, [line, B, P], [pcir, O, B]]
[inter, C, [line, B, Q], [pcir, O, B]]
```



**Example 131** The constraints:  $[\text{cong}, O, A, O, B]$ ,  $[\text{coll}, E, D, C]$ ,  $[\text{coll}, P, A, E]$ ,  $[\text{coll}, P, B, D]$ ,  $[\text{angle}, E, P, D, a1]$ ,  $[\text{cong}, O, A, O, E]$ ,  $[\text{cong}, O, A, O, D]$ . The following points and their construction order are given:  $A, B, O, C$ .

The construction sequence is

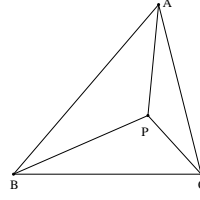
```
[point, A, B, C]
[on, O, [bline, A, B]]
[on, X0, [a_cir, A, B, a1]]
[inter, X1, [line, A, X0], [pcir, O, A]]
[inter, X2, [line, B, X0], [pcir, O, B]]
[mid, X3, X1, X2]
[inter, E, [ta_line, C, O, X3], [pcir, O, A]]
[inter, D, [line, E, C], [pcir, O, E]]
[inter, P, [line, A, E], [line, B, D]]
```



**Example 132** The constraints:  $[\text{eqangle}, C, B, P, A, C, P]$ ,  $[\text{eqangle}, B, A, P, A, C, P]$ . The following points and their construction order are given:  $A, B, C$ .

The construction sequence is

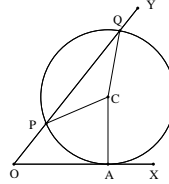
```
[point, A, B, C]
[inter, X0, [bline, B, C], [tline, C, C, A]]
[inter, X1, [bline, C, A], [tline, A, A, B]]
[inter, P, [pcir, X0, B], [pcir, X1, C]]
```



**Example 133** The constraints: [dis, C, A, d1], [dis, C, P, d1], [dis, C, Q, d1], [coll, A, O, X], [perp, C, A, O, X], [coll, P, O, Y], [coll, Q, O, Y], [angle, C, P, Y, a1]. The following points and their construction order are given: O, X, Y.

The construction sequence is

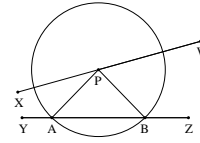
```
[point, O, X, Y]
[inter, X0, [nline, O, Y, a1], [rcir, O, d1]]
[inter, C, [d_line, O, X, d1], [pline, X0, O, Y]]
[foot, A, C, O, X]
[inter, P, [line, O, Y], [rcir, C, d1]]
[inter, Q, [line, P, O], [pcir, C, P]]
```



**Example 134** The constraints: [angle, A, P, B, a1], [coll, A, Y, Z], [coll, B, Y, Z], [coll, P, X, W], [dis, P, A, d1], [dis, P, B, d1]. The following points and their construction order are given: X, W, Y, Z.

The construction sequence is

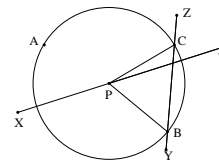
```
[point, X, W, Y, Z]
[inter, X0, [line, Y, Z], [rcir, Y, d1]]
[inter, X1, [nline, X0, Y, a1], [rcir, X0, d1]]
[inter, X2, [line, Y, Z], [pcir, Y, X1]]
[inter, X3, [rcir, X2, d1], [rcir, Y, d1]]
[inter, P, [line, X, W], [pline, X3, Y, Z]]
[inter, A, [line, Y, Z], [pline, P, Y, X3]]
[inter, B, [line, Y, Z], [pline, P, X2, X3]]
```



**Example 135** The constraints: [coll, P, X, W], [angle, B, P, C, a1], [coll, B, Y, Z], [coll, C, Y, Z], [cong, P, A, P, B], [cong, P, B, P, C]. The following points and their construction order are given: A, X, W, Y, Z.

The construction sequence is

```
[point, A, X, W, Y, Z]
[inter, X0, [line, W, X], [line, Y, Z]]
[on, X1, [nline, Y, Z, [dif, pi2, [div, a1, 2]]]]
[inter, X2, [line, Y, Z], [pline, W, Y, X1]]
```



[inter,  $X_3$ , [line,  $X_0$ ,  $A$ ], [pcir,  $W$ ,  $X_2$ ]]  
[inter,  $P$ , [line,  $X$ ,  $W$ ], [pline,  $A$ ,  $W$ ,  $X_3$ ]]  
[inter,  $B$ , [line,  $Y$ ,  $Z$ ], [pcir,  $P$ ,  $A$ ]]  
[inter,  $C$ , [line,  $B$ ,  $Y$ ], [pcir,  $P$ ,  $B$ ]]

### 3 Timing Statistics

The constraint solver is implemented using SB-Prolog on a Sparc-10 workstation. Table 1 contains the timing needed for the constraint solver (in interpreter mode) to find the first solution to the constraint problems. Table 2 contains some statistics on the timing. The average timing is 1.84.

Example	Time(s)	Example	Time(s)	Example	Time(s)	Example	Time(s)
2	0.38	3	5.27	4	0.35	5	0.7
6	1.01	7	2.71	8	1.39	9	0.83
10	0.2	11	0.51	12	1.08	13	0.37
14	0.61	15	0.28	16	0.61	17	1.41
18	0.4	19	1.47	20	0.38	21	0.26
22	9.56	23	0.28	24	0.78	25	2.91
26	0.65	27	1.11	28	0.56	29	1.04
30	0.43	31	2.06	32	0.87	33	0.33
34	0.64	35	0.59	36	0.62	37	1.23
38	1.15	39	0.69	40	0.8	41	1.74
42	3.41	43	3.1	44	1.31	45	1.99
46	0.64	47	4.84	48	0.74	49	5.2
50	1.21	51	21.42	52	11.59	53	0.31
54	0.6	55	0.69	56	0.77	57	3.23
58	0.24	59	0.36	60	0.2	61	1.11
62	0.63	63	0.49	64	1.94	65	0.85
66	0.53	67	0.96	68	1.51	69	2.29
70	2.77	71	5.18	72	1.21	73	3.06
74	2.36	75	0.93	76	1.68	77	1.36
78	0.31	79	0.38	80	0.88	81	0.94
82	1.52	83	1.0	84	8.29	85	0.59
86	2.02	87	0.78	88	0.66	89	2.98
90	1.64	91	1.22	92	3.05	93	1.61
94	1.62	95	0.43	96	0.45	97	0.58
98	0.74	99	0.76	100	0.4	101	1.68
102	0.38	103	3.01	104	0.84	105	0.57
106	2.17	107	18.63	108	1.11	109	1.44
110	3.2	111	1.47	112	2.97	113	4.72
114	2.98	115	1.66	116	1.43	117	4.53
118	4.97	119	1.72	120	1.0	121	0.87
122	0.5	123	0.35	124	1.71	125	0.87
126	1.09	127	1.08	128	1.05	129	1.11
130	0.92	131	1.32	132	0.3	133	0.68
134	1.45	135	1.16				

Table 1. Timing for the Constraint Problems